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ANALYSIS OF FLOW IN NETWORKS OF CONDUITS OR CONDUCTORS

BY

HARDY CROSS

BULLETIN No. 286

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ENGINEERING EXPERIMENT STATION

BULLETIN NO. 286 NOVEMBER, 1936

ANALYSIS OF FLOW IN NETWORKS OF CONDUITS OR CONDUCTORS

BY

HARDY CROSS
PROFESSOR OF STRUCTURAL ENGINEERING

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SYNOPSIS

The problem of finding the distribution of flow in networks of pipes occurs in the design of distribution systems for water. Similar problems occur in connection with distribution systems for other fluids such as steam or air and with electrical circuits. In general the analysis of such systems by formal algebraic procedures is, if the systems are complicated, very difficult. Models have been used in studying this problem, but here, as elsewhere, there are some objections to their use.

The methods here presented are methods of successive corrections. The convergence is apparently sufficiently rapid in all cases to make the methods useful in office practice. Perhaps the greatest value of the methods is in training and assisting the judgment, but this, of course, is the principal object of all analytical procedures.

Where the relation between flow and head (between current and voltage) is linear, the corrections are exact. Where this relation between flow and head is not linear, use is made of the relation between increment of flow and increment of head, which relation is linear for a given quantity of flow. If, however, the increments are fairly large, this linear relation is somewhat in error. A sufficiently exact solution is nevertheless obtained by successive correction. The method thus involves an arithmetical application of the fundamental principle of the differential calculus.

In the problems of flow discussed in detail in this bulletin it is assumed that kinetic heads and head losses at junctions are small in comparison with the lead losses due to friction, and that they may therefore be neglected.

The problems under immediate discussion deal primarily with systems carrying water and incidentally with those carrying electric currents. Additional applications of the methods here presented are suggested at the end of the paper.

Perhaps it should be added that this investigation, in a field of study which is not the major interest of the author, is a by-product of explorations in structural analysis.

The problems have been chosen to illustrate the method and not because they represent any existing layouts.

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ANALYSIS OF FLOW IN NETWORKS OF CONDUITS OR CONDUCTORS

I. Introduction

1. *Type of Problem.*—As indicated later, various types of problems occur in connection with flow in networks. In what is perhaps its simplest form, the problem is usually as follows:

The quantity of fluid or of energy flowing into the system at one point is known and the point of delivery is also known. The sizes and lengths of the conductors or pipes in the system are given or assumed, and also the law controlling the relation between quantity of flow in the conductor and the loss of head or voltage in a given length.

It is usually desired to determine the total loss of head or voltage between inlet and outlet. If a single conductor connected these two points, the loss of head for given flow could be computed directly from the relation between flow and head loss. In a network, however, this loss depends on the distribution of the flow in the system. If such distribution is known, the drop of potential in each conductor can be determined directly, and the total drop found as the sum of the drops along any path connecting inlet and outlet, the total drop being of course the same whatever path is chosen.

The difficulty arises in determining the distribution of flow in the network. This is controlled by two sets of conditions, both simple and obvious:

(a) The total flow reaching any junction equals the total flow leaving it (continuity of flow)

(b) The total change in potential along any closed path is zero (continuity of potential).

These sets of conditions, together with the relation between flow and potential drop, lead to sets of equations in which either the flows in the individual conductors or the potentials at the junction points are taken as the unknowns.

If the flows are taken as the unknowns, the equations will be those for continuity of potential; if the potentials are the unknowns, the equations will be those for continuity of flow.* In either case,

*The two methods here presented represent general methods applicable to many engineering problems. The physical conditions controlling engineering relations often consist of two groups of laws which are quite independent of each other. In such cases either set of relations may be first expressed in terms of the other, and then a formal or an approximate solution may be obtained to satisfy the second condition. Compare the analysis of continuous frames by methods such as the theorem of three moments, where the equations to be solved represent the geometrical relations, the unknowns having been previously interrelated by statics, with the method of slope-deflection where the equations are those of statics, the unknowns having been previously related from geometrical considerations.

the order of the equations will be that of the relation between flow and loss of potential. If this relation is linear, the equations will be linear. If, however, the relation is not linear, serious difficulties arise in solving the equations.

In those cases where the relation between flow and change of head is linear, the methods to be presented may be thought of as a bookkeeping procedure for solving linear equations; where the relation is not linear, the method changes the problem into that of a succession of linear relations by use of the fundamental principle of the differential calculus.

2. *Flow of Water in a Network of Pipes.*—In any network of pipes such as is shown in the problems discussed in the following pages, it is known that in each closed circuit the sum of all changes in head is zero, and that at each junction the quantity flowing into the junction equals the quantity flowing away from the junction.

It is assumed further that we know the law determining the loss of head in any length of pipe for a given flow. This law usually takes the form

$$h = CV^n$$

where h is the change in head accompanying flow in any length of pipe, C is the loss in the pipe for unit velocity of flow, and V is the velocity.

Since the quantity of water flowing in the pipe is AV, this relation may be rewritten

$$h = rQ^n$$

where r is the loss of head in the pipe for unit quantity of flow. The quantity r depends on the length and diameter of pipe and on its roughness.

The problem is to find the amount of water flowing in each pipe. When the distribution of flow is known, the losses of pressure throughout the system are readily computed.*

It is important to note that, except as noted in the footnote, only relative values of r are needed to determine distribution of flow.

II. Methods of Analysis

3. *Methods of Analysis Proposed.*—Two methods of analysis are proposed. In one of these the flows in the pipes or conductors of

*In ordinary cases (on the assumption $h = rQ^n$ with n the same for all pipes) the distribution of total flow is independent of the quantity flowing. On certain assumptions as to the relation between head and flow, this will not be true. The matter does not seem of immediate practical importance, though it may be of scientific interest in some cases.

the network always satisfy the condition that the total flow into and out of each junction is zero, and these flows are successively corrected to satisfy the condition of zero total change of head around each circuit. In the other method the total change of head around each circuit always equals zero, and the flows in the pipes of the circuit are successively adjusted so that the total flow into and out of each junction finally approaches or becomes zero.

The former method is, for convenience of reference, here designated as the "Method of Balancing Heads," the latter as the "Method of Balancing Flows." The method of balancing heads is directly applicable where the quantities flowing at inlets and outlets are known. The method of balancing flows is directly applicable when the heads at inlets and outlets are known; in this case it will probably be found more convenient than that of balancing heads. In some problems it may be desirable to combine the two methods.

Both methods depend on the principle that the resistance to change in flow in any pipe equals approximately $nrQ^{(n-1)}$ where $h = rQ^n$.

III. Method of Balancing Heads

4. *Statement of Method.*—The method of solution is as follows:

(a) Assume any distribution of flow.

(b) Compute in each pipe the loss of head $h = rQ^n$. With due attention to sign (direction of potential drop), compute the total head loss around each elementary closed circuit $\Sigma h = \Sigma rQ^n$.

(c) Compute also in each such closed circuit the sum of the quantities $R = nrQ^{(n-1)}$ without reference to sign.

(d) Set up in each circuit a counterbalancing flow to balance the head in that circuit (to make $\Sigma rQ^n = 0$) equal to

$$\Delta = \frac{\Sigma rQ^n \text{ (with due attention to direction of flow)}}{\Sigma nrQ^{(n-1)} \text{ (without reference to direction of flow)}}$$

(e) Compute the revised flows and repeat the procedure.

Continue to any desired precision.

In applying the method it is recommended that successive computations of the circuits be put on identical diagrams of the system. In office practice such diagrams will usually be white prints. Write in each elementary circuit the value ΣR, and outside the circuit write first (above) the value Σh for flow in a clockwise direction around the circuit and second (below) the value Σh for flow in a counterclockwise direction around the circuit. On the right of these figures put an arrow pointing ⤵ or ⤴ to the larger figure. This arrow will show

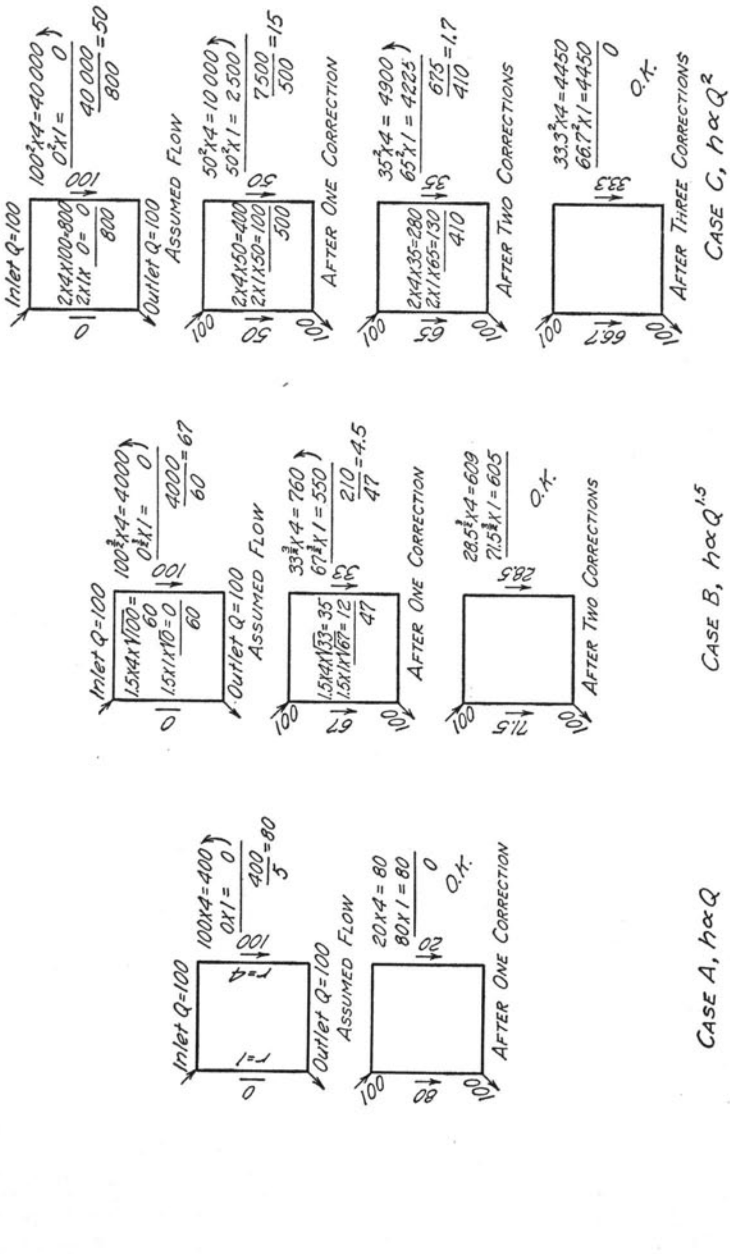


FIG. 1. DISTRIBUTION OF FLOW IN SINGLE CIRCUIT; METHOD OF BALANCING HEADS

correctly the direction of counterflow in the circuit. This technique is illustrated in the problems following.

5. *Proof of Method.*—If the distribution of flow assumed in the first place were correct, the change of head around any single closed circuit would be zero. This change of head is ΣrQ^n. Considering for the present a single circuit, write for each pipe

$$Q = Q_0 + \Delta$$

Then, $$rQ^n = r(Q_0 + \Delta)^n = r(Q_0^n + nQ_0^{(n-1)}\,\Delta + \quad)$$

If Δ is small compared with Q_0 the remaining terms in the expansion may be neglected.

Then, for

$$\Sigma rQ^n = 0$$
$$\Sigma rQ_0^n = -\Delta_0 \Sigma nrQ_0^{(n-1)}$$
$$\Delta = -\frac{\Sigma h}{\Sigma R} = -\frac{\Sigma rQ_0^n}{\Sigma nrQ_0^{(n-1)}}$$

Of course, if Δ is relatively large compared with Q_0 and n is greater than unity, the approximation is not very good, but this is less important than it might at first seem, because in any case we must correct for the unbalanced head produced in one circuit by corrections in the adjacent circuits, which in general requires a recomputation of all circuits. The convergence is, for practical purposes, sufficiently rapid.

6. *Illustrative Problems.*—

Problem 1.—Single Closed Circuit
(Figure 1)

This problem shows the elementary procedure for a single circuit in each of three cases: (a) where h varies as Q (streamline flow, or electrical resistance, E varies as I); (b) where h varies as $Q^{1.5}$ merely as an illustration of a fractional exponent; and (c) where h varies as Q^2, a common approximate value for water circuits.

In all of these cases it is required to distribute a flow of 100 between two pipes, one of which is four times as long as the other, but which are otherwise alike. In each case, also, to show the convergence, the first assumption is the worst possible, namely that the total flow follows the longer path.

Of course each of these cases is readily solved directly. Thus, in all cases let Q_1 be the flow in the shorter, and Q_2 that in the longer pipe. Then

$$\text{(a)}\quad \frac{Q_1}{Q_2} = \frac{4}{1} \qquad Q_1 = \frac{4}{5} \cdot 100 = 80$$

$$\text{(b)}\quad \frac{Q_1}{Q_2} = \frac{(4)^{2/3}}{(1)^{2/3}} \qquad Q_1 = \frac{2.52}{3.52} \cdot 100 = 71.5$$

$$\text{(c)}\quad \frac{Q_1}{Q_2} = \frac{(4)^{1/2}}{(1)^{1/2}} \qquad Q_1 = \frac{2}{3} \cdot 100 = 66.7$$

The computations have in these cases been arranged in the order explained already, which has been found very convenient, namely,

(1) Write the divisor $\Sigma nrQ^{(n-1)}$ within the circuit.

(2) Write on one side of the circuit first the sum ΣrQ^n for clockwise flow, and below this the sum ΣrQ^n for counterclockwise flow. If, then, the arrow indicating the direction of flow is written on the right of these figures and pointing to the larger flow, it will correctly indicate the direction of counterflow needed to balance the circuit. In complicated problems observance of some such system is necessary.

Problem 2.—Simple Network—h Varies as Q^2

(Figure 2)

This shows a very simple network with one inlet and one outlet. It is here assumed that h varies as Q^2. All pipes are assumed to be alike. The purpose of the problem is to illustrate the method of arrangement in a case slightly more difficult than that of a single circuit.

Here, as before, a very bad first trial value was intentionally chosen to illustrate the procedure. Of course, the exact solution is at once known by inspection to be as shown, and almost any reasonable trial converges rapidly.

The arrangement of computations is that previously recommended. Note the relatively small change in $\Sigma nQ^{(n-1)}$.

Procedure

Some distribution of flow (without excesses or deficiencies at the junctions) is first assumed.

(1) Compute the unbalanced heads around each circuit. (In this case $h = Q^2$ in each pipe.) As previously noted, these are written to one side of the circuit, first (above) the heads for clockwise flow within that circuit, next (below) those for counter clock-

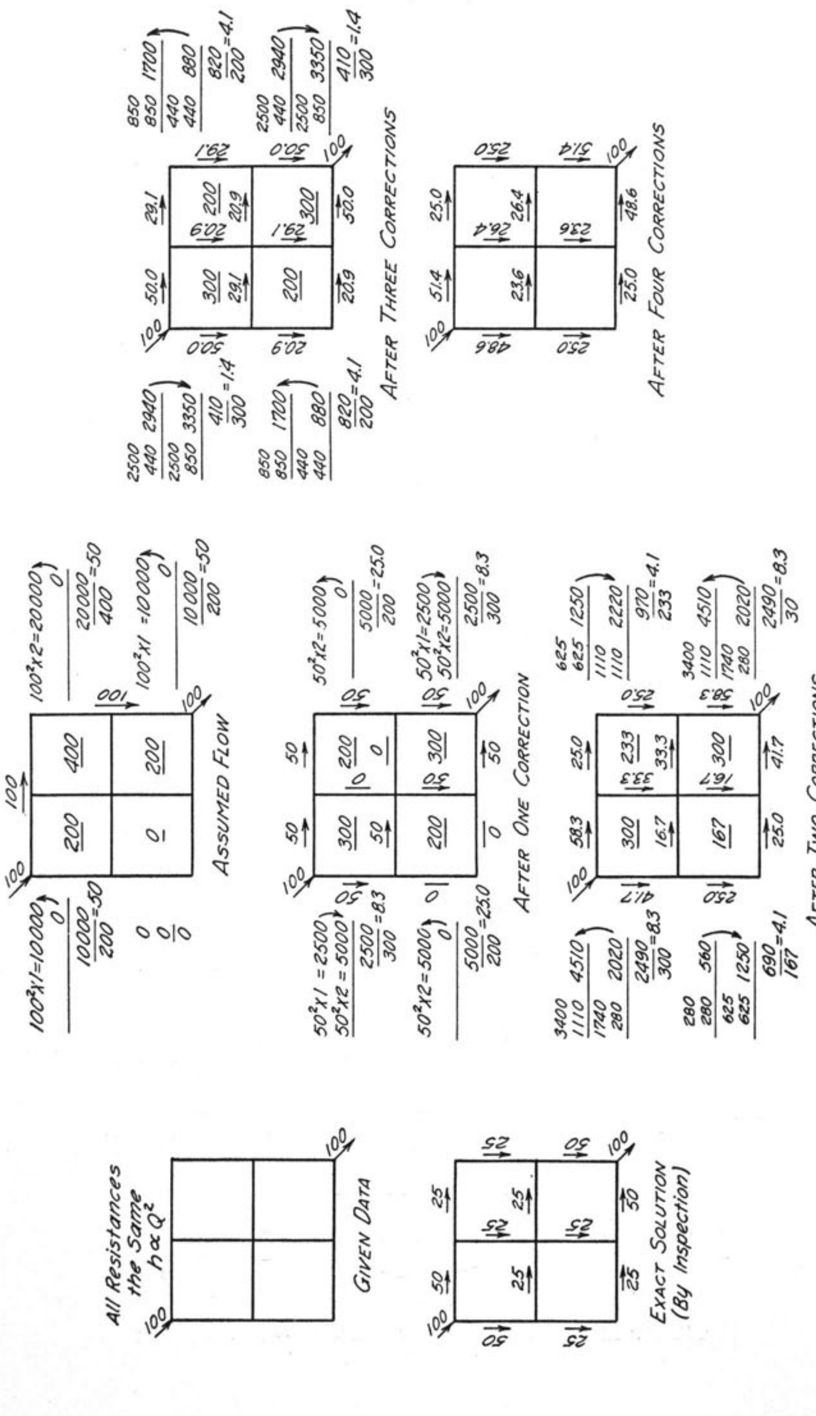


Fig. 2. Distribution of Flow in Simple Network; Method of Balancing Heads

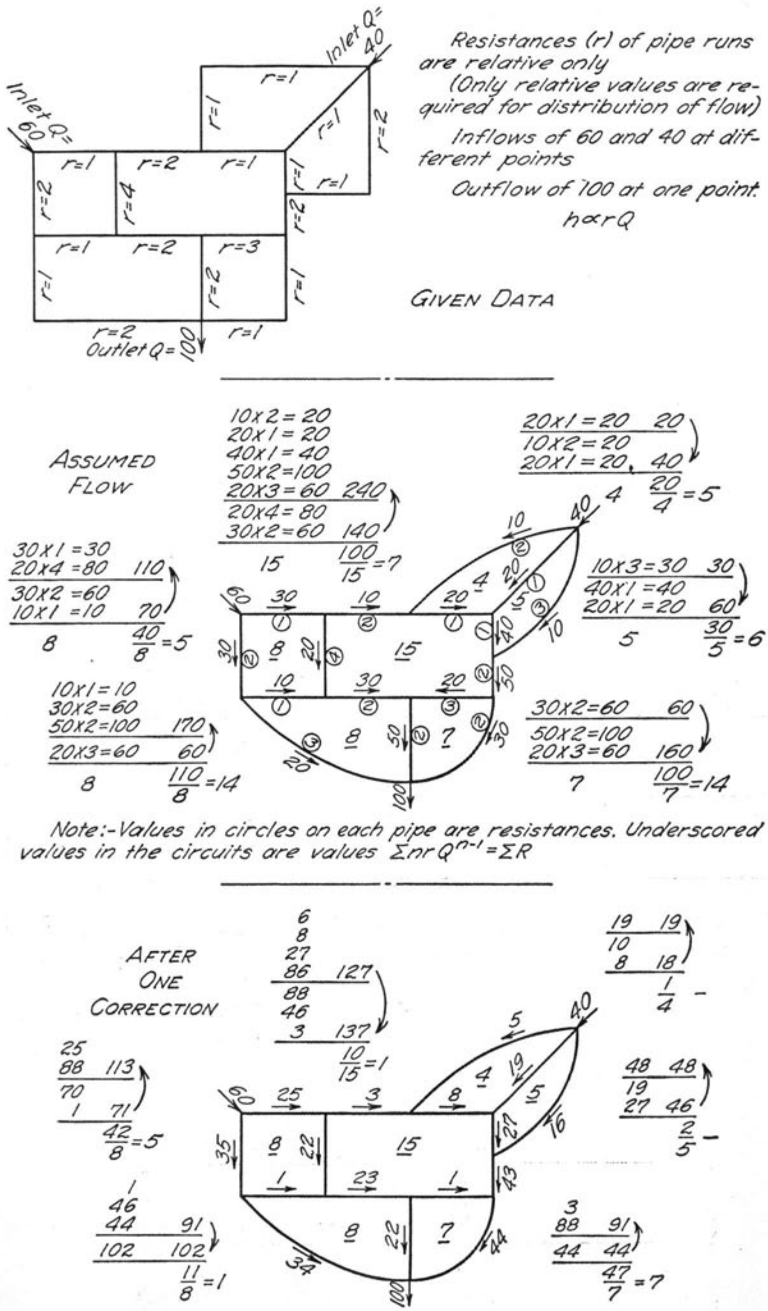


FIG. 3. DISTRIBUTION OF FLOW IN NETWORK—TWO INLETS; METHOD OF BALANCING HEADS

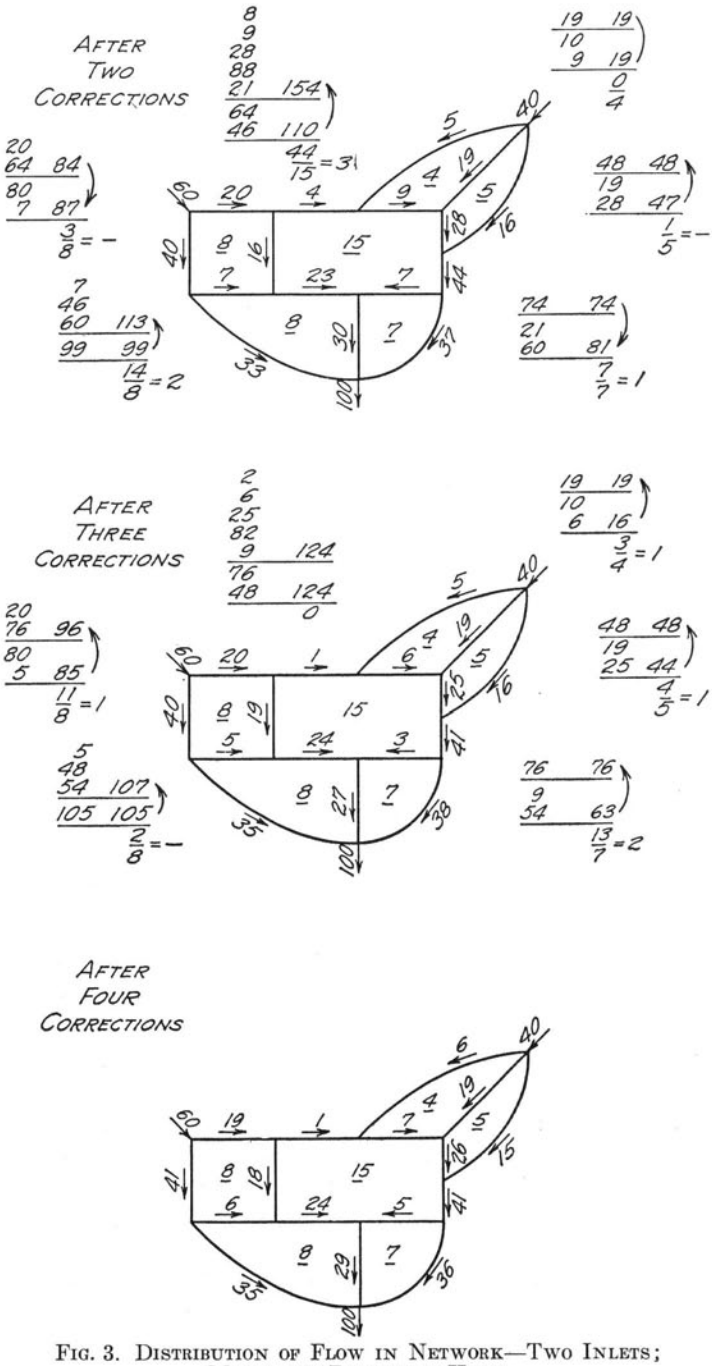


FIG. 3. DISTRIBUTION OF FLOW IN NETWORK—TWO INLETS; METHOD OF BALANCING HEADS

wise flow. The difference is the unbalanced head. The arrow, ↺ or ↻, written to the right of these figures, points to the larger head loss, and indicates the direction of counterflow.

(2) Compute the divisors $\Sigma nrQ^{(n-1)}$. In this case $nrQ^{(n-1)} = 2Q$ in each pipe, since $r = 1$. These are written within the circuits.

(3) Revise the flows from these counterflows, and repeat the process as often as required.

In the foregoing solution the method is intentionally applied blindly, without any judgment at all. It is at once evident that all circuits in the first trial are badly unbalanced, all flows in each circuit being in one direction. Usually the circuits which are badly unbalanced will at once be thrown more nearly into balance by guess.

Problem 3.—Complex Network—Two Inlets—h Varies as Q
(Figure 3)

This is a more complicated network, with two inlets and one outlet. The solution involves no new principles. Here it is assumed that h varies as Q. Of course, in this case the divisors for each circuit are constant throughout.

No attempt has been made to compute fractional values. It is, however, clearly possible to go to any reasonable degree of precision, but with considerable increase in the computation required if results correct to three figures are wanted.

The arrangement of computations follows that previously explained.

Problems 4 and 5.—Systems of Pipes in Different Planes Interconnected
(Figures 4 and 5)

In general, systems for distributing water in cities may, for purposes of analysis, be considered as in a single plane. In other cases, as, for example, in distributing steam or hot water to a heating system, the distribution may take place in several planes, with interconnection between the planar systems of distribution.

This type of problem presents no especially new features except that successive distribution must be made in circuits closed by the risers as well as in the circuits which lie in a plane. The pipes chosen on each floor to close the circuits containing the risers are selected arbitrarily. It will be noted that in such problems any pipe may lie in only one circuit (an outside pipe in a floor) or in two circuits, three circuits, or even in four circuits (two floor circuits and two riser circuits). The total change in flow in the pipe is the sum of the changes in all the circuits of which it is a member.

Problem 4 shows a rather impractical layout selected for simplicity of illustration. The distribution is carried through only two steps to show the procedure.

Problem 5 differs from Problem 4 only in having more risers.

Clearly the technique used in recording the flows is a matter of individual choice. Some may prefer to use isometric diagrams throughout the analysis.

It is believed that the diagrams are self-explanatory.

7. *Characteristics of Procedure.*—Certain characteristics of the procedure will be noted. When the flow is adjusted in any circuit the flow is increased in some pipes and decreased in others, so that the quantity $\Sigma R = \Sigma nrQ^{(n-1)}$ is not very much changed, and need not usually be recomputed for each change of flow.

Since the first adjustments are in a sense preliminary, it is useless to attempt precision in making them.

The answer, when finally obtained, is inevitably correct, since it satisfies the conditions that the quantities balance at each junction, and that the heads balance around each circuit. Moreover, errors in the procedure are not cumulative, and, if made, are ultimately eliminated.

IV. Method of Balancing Flows

8. *Statement of Method.*—In the method of analysis by balancing heads just presented, the flow at any junction is balanced throughout the analysis, but the head around any circuit is balanced by successive correction.

Another method is to keep the head balanced around any circuit throughout the analysis, in which case the flow at the junctions is balanced by successive correction.

We may, then, assume a series of heads throughout the system and compute the flow in each pipe corresponding to the differences of head. From these find the total flow to each junction except inlets and outlets. Distribute this flow to the pipes connecting at the junction in inverse proportion to their resistances. ($R = nrQ^{(n-1)}$). This, of course, causes an excess (or deficiency) of flow at the next junction, but by successive distribution the excess flow will ultimately be squeezed out at the inlets and outlets of the system.

Note, in the first place, that if the flow is distributed as in the

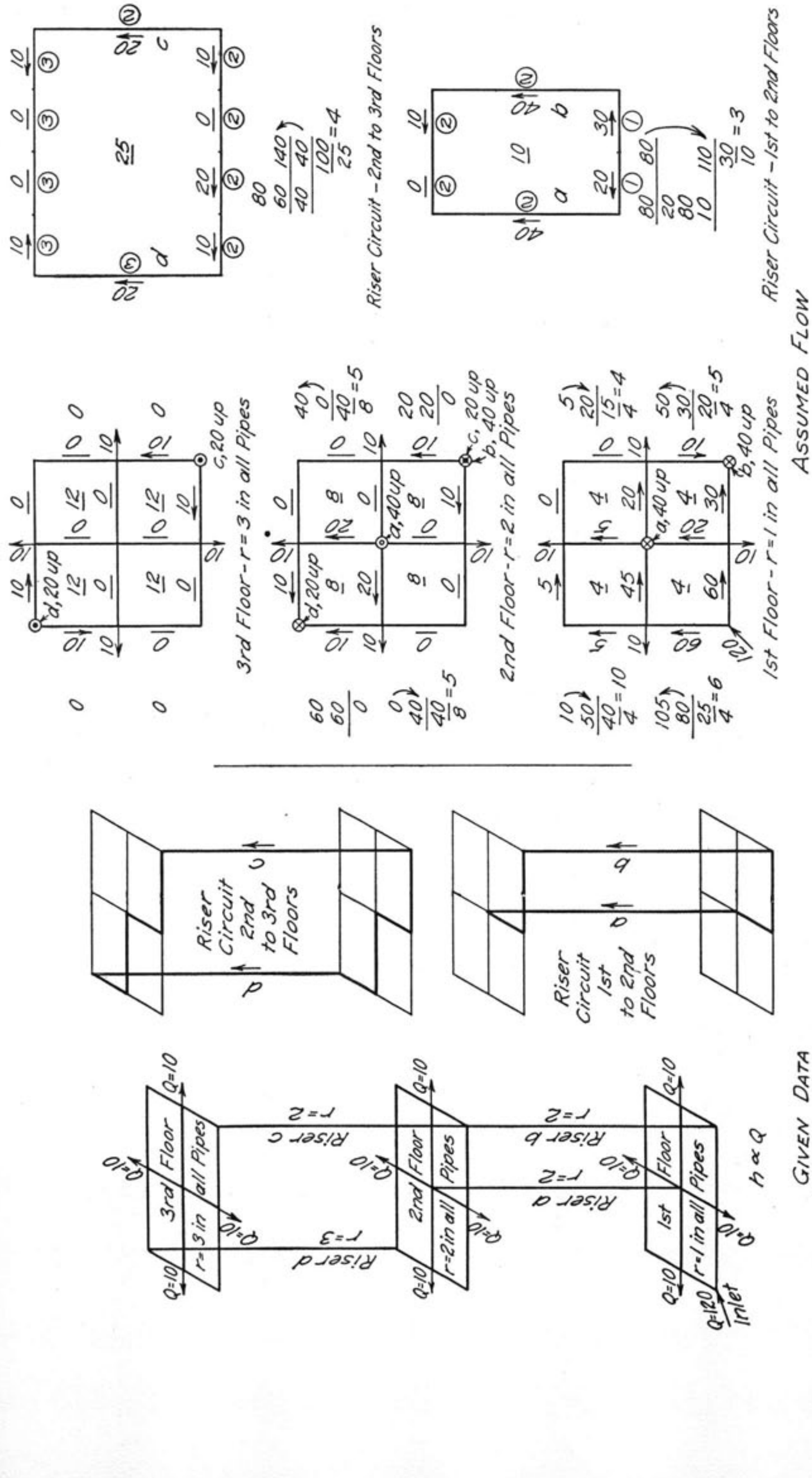


FIG. 4. DISTRIBUTION OF FLOW IN SEVERAL PLANES; METHOD OF BALANCING HEADS

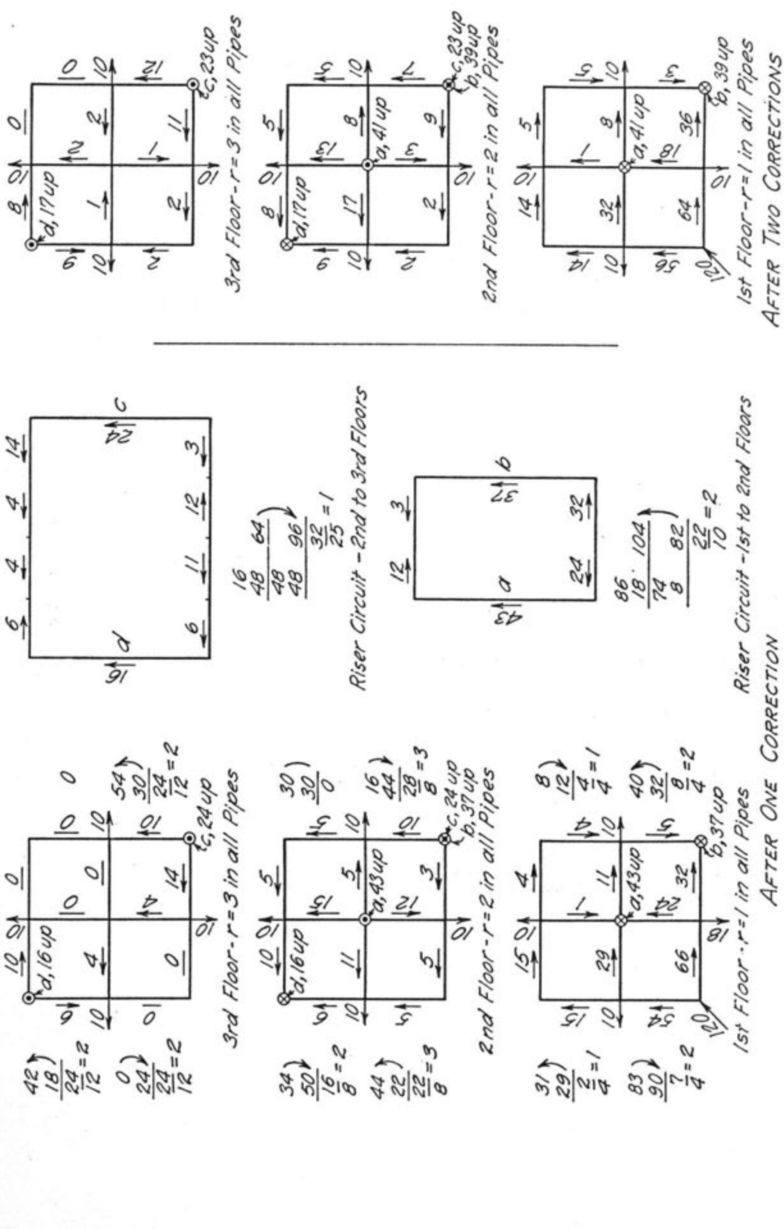
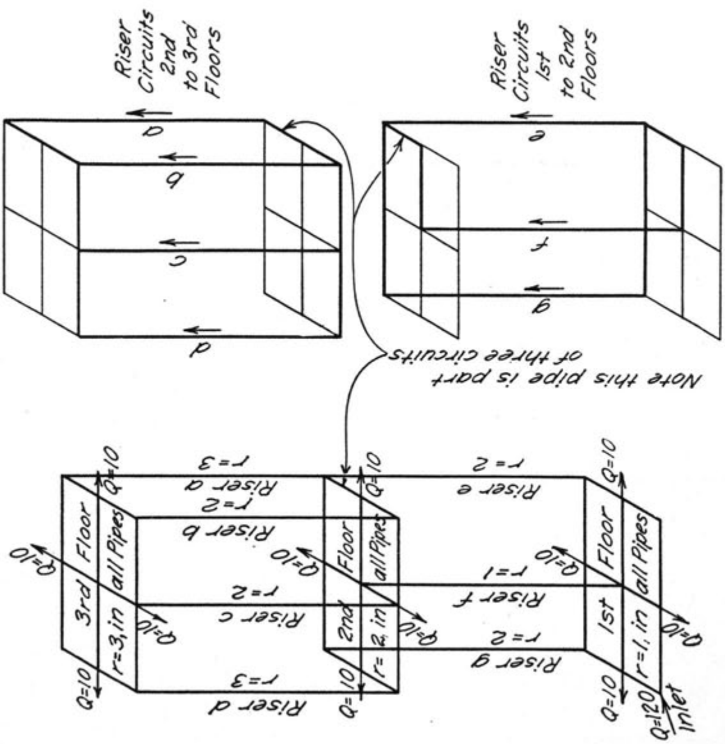


FIG. 4. DISTRIBUTION OF FLOW IN SEVERAL PLANES; METHOD OF BALANCING HEADS



GIVEN DATA

FIG. 5. DISTRIBUTION OF FLOW IN SEVERAL PLANES; METHOD OF BALANCING HEADS

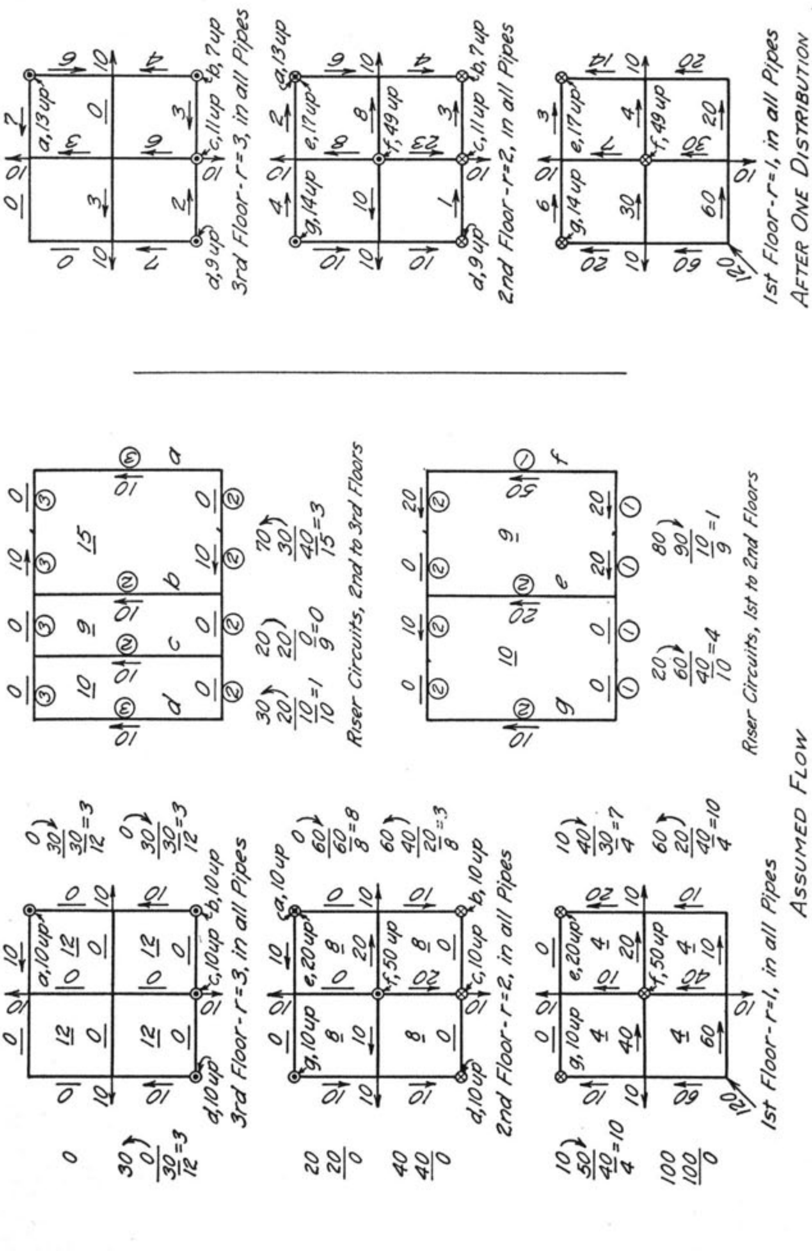


Fig. 5. Distribution of Flow in Several Planes; Method of Balancing Heads

foregoing the head difference between any inlet and outlet is approximately unchanged, the change of head in one direction along one part of any path connecting two openings being equal and opposite to the change of head along another part. Of course, the values $R = nrQ^{(n-1)}$ are not exact when n is greater than unity, because the values of Q are not exact. Hence revision of the values of R is necessary.

Note also that the total flow into and out of the system is in general not balanced in the beginning, but that any excess of inflow will untimately be squeezed out by successive distributions. Similarly, if there is a deficiency, we may imagine that it will inflow at the inlets and outlets.

9. *Illustrative Problems.*—

Problem 6.—Simple Network—h Varies as Q^2
(Figure 6)

The problem analyzed in Fig. 6 is the same as that previously analyzed by balancing heads in Fig. 2. The elevation at the inlet is assumed to be 100. The outflow takes place at one point as shown. It is assumed that in each pipe-run of the system the loss of head equals the square of the quantity flowing. The procedure is as follows:

Procedure

(1) Assume heads arbitrarily for inlet and outlet. In this case assume inlet at El. 100 and outlet at El. 0. Guess at heads at other junctions as closely as possible. These values are shown in circles near the junctions.

(2) Compute flow (or relative flow) in each pipe for heads assumed. In this case it is assumed that $h = rQ^2$, and therefore that $Q = \sqrt{\frac{h}{r}}$.

(3) Compute excess (+) or deficiency (−) of flow at each junction except inlet and outlet. Write these in the squares shown for each junction.

(4) Distribute these excesses or deficiencies to the pipes connecting at each junction in inverse proportion to $R = nrQ^{(n-1)}$. In this problem all values of r are assumed equal to unity, except in the corner pipes which are twice as long, and hence have $r = 2$. Also $n = 2$. Hence the flows are distributed in proportion to $\frac{1}{rQ}$, where Q is the flow already computed.

(5) Carry over in the pipes connecting at each junction (except inlet and outlet) the flows distributed from adjacent junctions to

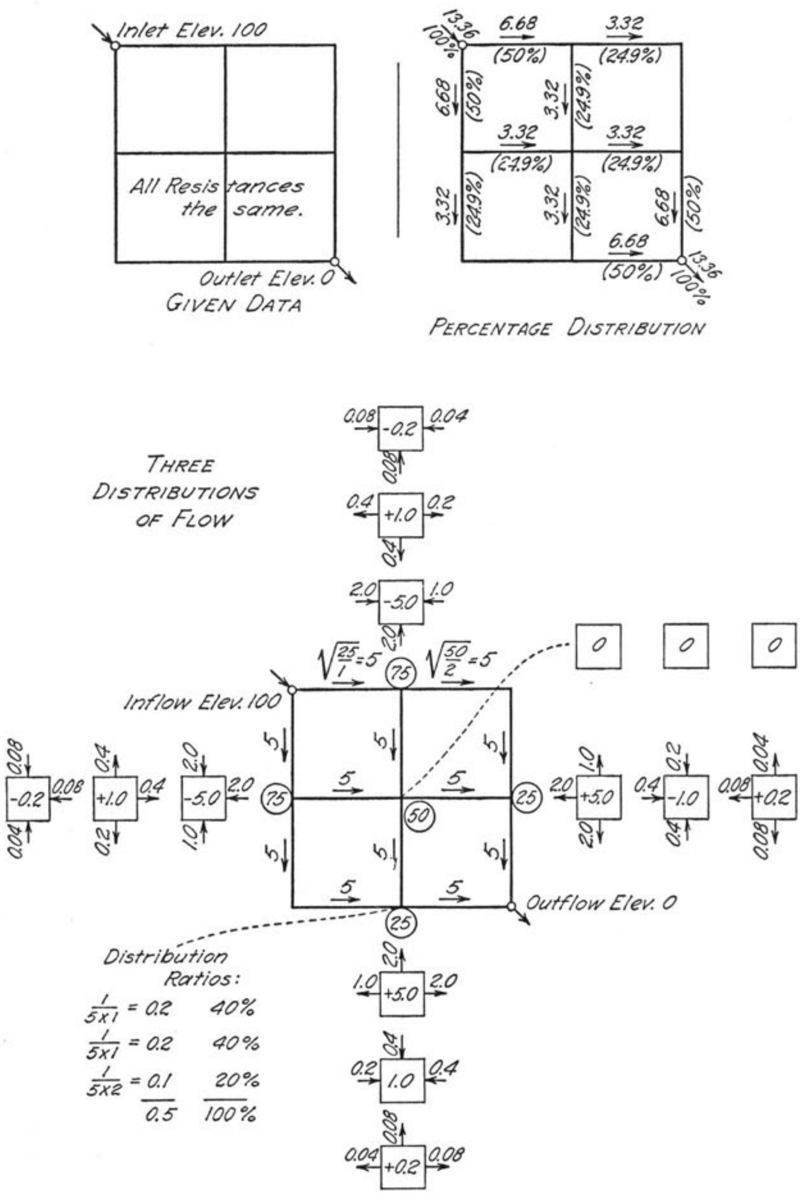


FIG. 6. DISTRIBUTION OF FLOW IN SIMPLE NETWORK; METHOD OF BALANCING FLOWS

find new values of excesses and deficiencies.

(6) Distribute the new unbalanced flows and repeat to convergence, thus determining the relative amount of the total inflow flowing in each pipe.

Problem 7.—Multiple Inlets and Outlets
(Figure 7)

In this problem it is assumed that the elevations are known at several inlets—namely, El. 40 at inlet A, and El. 30 at inlet B—and at several outlets—namely, El. 10 at outlets B and D, and El. 0 at outlets A and C. It is desired to determine the distribution of total flow at inlets and at outlets, and, in general, the distribution of flow in the system. For simplicity of presentation it is assumed that the resistances are the same in all pipe-runs. It is also assumed that $h = \dfrac{Q^2}{1000}$ in any pipe.

First assume heads at intermediate junctions as shown by figures in the circles at each intermediate junction.

Compute flows—or relative flows—in each pipe. Here the flow is computed as $Q = \sqrt{1000h}$.

Compute excess or deficiency of flow at each junction, and write this in the rectangle near that junction. Distribute these unbalanced flows in proportion to $\dfrac{1}{Q}$ at each junction. The relative values for distribution at each junction are written on the pipes at that junction.

Compute the excesses and deficiencies produced by the new flows and distribute these.

It may be assumed that in a problem of this type what is really wanted is the distribution of outflow and of inflow. Table 1 shows these flows after successive distributions in this case. The convergence here is slow and not very satisfactory, partly because the first guess as to intermediate heads is not very good. Nevertheless, the computations indicate quickly the approximate distribution of flow at inlets and outlets.

Some circuits will be found to be unbalanced as regards head. These may now be adjusted, if desired, starting with the distribution found for total flow.

10. *Remarks on Method.*—In such relatively complex problems as that just shown, it seems clear that the method of distributing flows has advantages. In general, however, it is thought to be less simple, obvious, and expeditious than that of balancing heads.

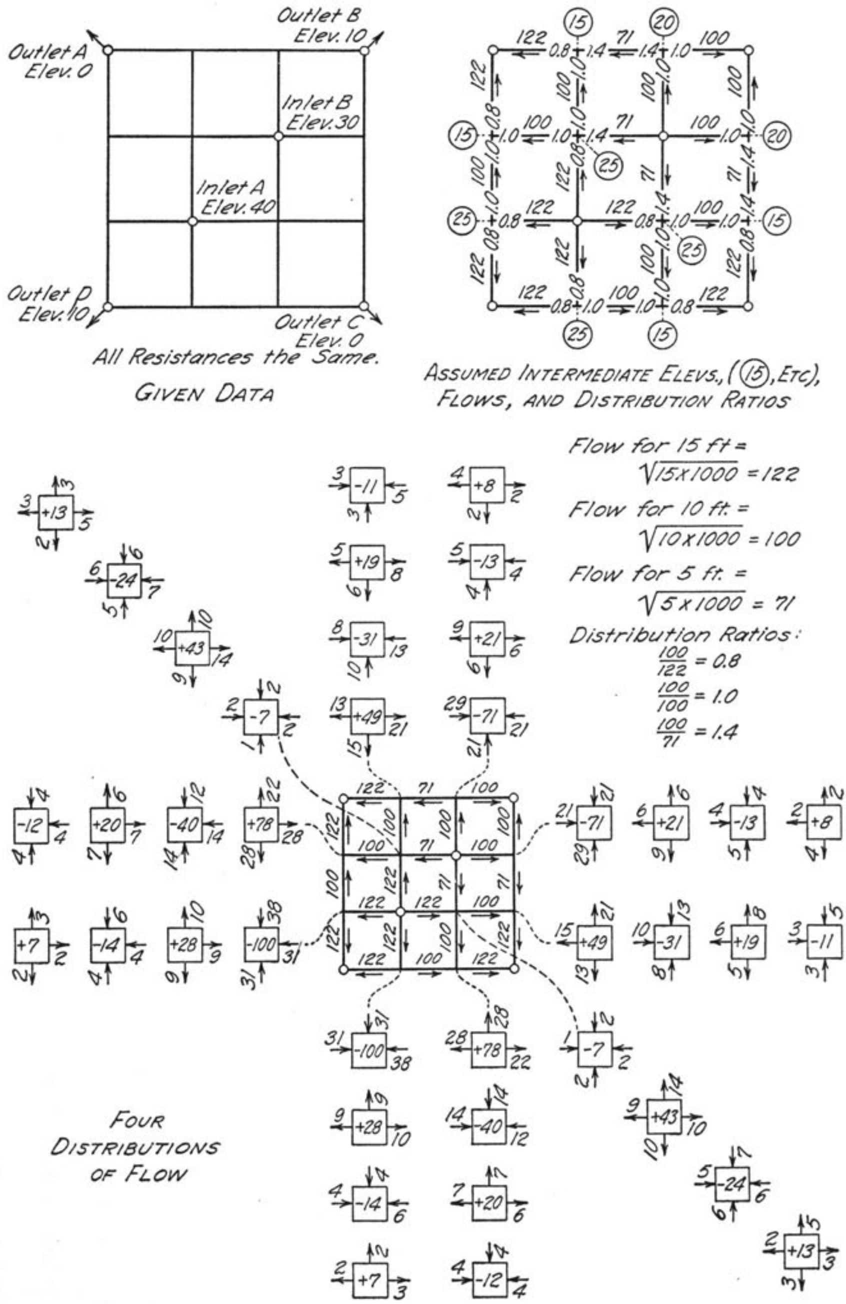


FIG. 7. DISTRIBUTION OF FLOW IN NETWORK—SEVERAL INLETS AND OUTLETS; METHOD OF BALANCING FLOWS

TABLE 1
PROBLEM 7.—SUCCESSIVE VALUES OF INFLOW AND OUTFLOW

	Assumed Values	Values After One Distribution	Values After Two Distributions	Values After Three Distributions	Values After Four Distributions	Estimated True Values	
						Value	Per Cent
Inlet A	488	552	525	543	535	537	59
Inlet B	342	388	348	370	356	361	41
Total Inflow	830	940	873	913	891	898	100
Outlet A	244	279	259	270	263	268	30
Outlet B	200	158	170	162	166	166	18
Outlet C	244	279	259	270	263	268	30
Outlet D	244	182	200	192	196	196	22
Total Outflow	932	898	888	894	888	898	100

V. TYPES OF PROBLEMS ENCOUNTERED IN NETWORKS

11. *Typical Problems.*—In systems carrying water, and in other networks of circuits, the problems may take various forms.

(a) Figure 8a. Given the inflow at one point and the point of discharge, to determine the distribution of flow and variation of head within the network. This problem represents the most elementary application of the method of balancing heads. (Problems 1, 2, 4, 5). It may also be solved by balancing flows. (Problem 6).

(b) Figure 8b. Given the distribution of inflow at several inlets and the distribution of outflow at several outlets, to determine the distribution of flow and the variation of head. (Problem 3). The problem is practically the same as that just discussed. It is more conveniently solved by balancing heads than by balancing flows.

(c) Figure 8c. Given the heads at various points of inflow or outflow, to determine the total flow and its distribution. Such a problem may occur in studying flow between reservoir systems. This may be solved directly by the method of balancing flows. (Problems 6, 7).

(d) Figure 8d. Given the heads at various points of inflow and the relation of head to flow at various points of outflow. This problem may be reduced to the problem just stated by imagining that at each outflow point an additional pipe or conductor is connected. This imaginary discharge pipe is to have such a resistance that for unit flow the loss of head would be the same as for unit flow through the outlet.

(e) Figure 8e. Given the heads at certain points of inflow or outflow and also the flow, independent of head, at other points, to

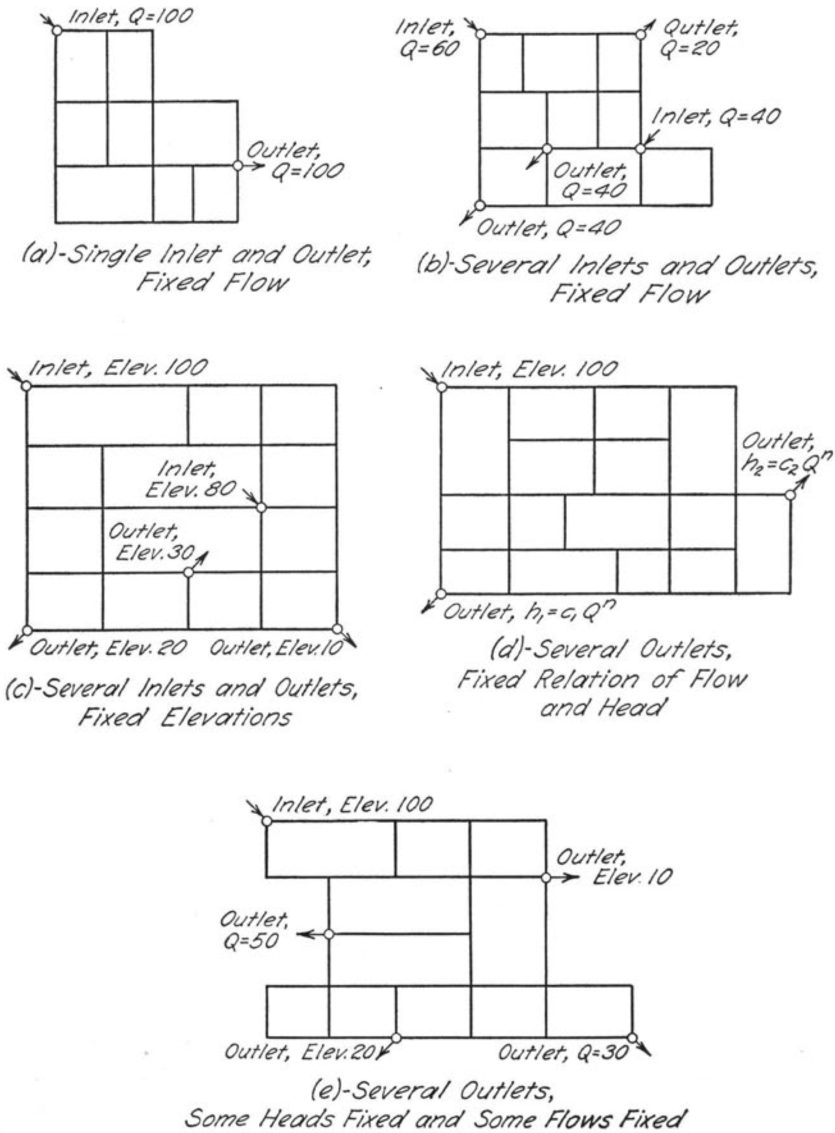


FIG. 8. TYPES OF PROBLEMS ENCOUNTERED IN DISTRIBUTING FLOW IN NETWORKS

determine the distribution of flow. This problem is essentially the same as that of fixed heads in (c). At the points of fixed flow allowance for the flow is to be made in computing excess or deficiency of flow. Moreover, in applying the method of distributing flows, the points of fixed flow are not to be considered as openings at which water may be squeezed out or sucked into the system.

Evidently various combinations of these problems may occur.

All the problems just stated deal with constant conditions of flow. The heads in the reservoirs, for example, are assumed to remain constant in spite of the flow. A common problem, however, is that in which the flow affects the heads as the flow continues, and it is desired to study the changes in flow conditions with time. The present paper makes no direct contribution to the solution of this problem, except as it indicates methods of solution for fixed conditions. However, such problems may be conveniently studied by the method of increments, thus:

(1) Compute the flow conditions for the initial heads.

(2) Compute from inflows and outflows the changes in head in any convenient short interval of time.

(3) Compute the *changes* of flow for these changes of head. This procedure, while approximate, is quite direct.

(4) Repeat the procedure.

VI. Concluding Remarks—Other Applications of Methods

As previously stated, this monograph deals primarily with flow of water in networks of pipes. The problems are restricted to cases in which the relation between loss of head and quantity of flow is of the relatively simple form h varies as Q^n. It is sometimes proposed that the flow of liquids may be controlled by a relation of the form $h = aQ + bQ^2$. In this case we can deduce $\frac{\Delta h}{\Delta Q} = (a + 2bQ)$, and the counterflow required to balance the head in any circuit becomes

$$\Delta Q = -\frac{\Sigma\,(aQ + bQ^2) \text{ with due attention to direction of flow}}{\Sigma\,(a + 2bQ) \text{ without reference to direction of flow}}$$

In more general terms, if $h = f(Q)$, then $\frac{\Delta h}{\Delta Q} = f'(Q)$ and the counterflow

$$\Delta Q = -\frac{\Sigma f(Q) \text{ with due attention to direction of flow}}{\Sigma f'(Q) \text{ without reference to direction of flow}}$$

This suggests that such arithmetical application of the principle of the differential calculus may have application in the solution of certain problems in simultaneous equations, but no effort has been made to explore this field.

Applications of the methods in the study of electrical circuits are evident.

This paper deals directly only with networks of definite conductors. Clearly, however, the methods may also be applied to study distribution of flow in those cases where no pipes really exist, but where the flow is diffused, as in cases of percolation through earthen dams or through soil strata where the operation of wells is to be investigated. In such cases flow in a series of imaginary pipes is substituted for the diffused flow. This leads, of course to the "flow net" picture. The principal value of the method in this connection is in checking and revising an assumed flow net, the pipes being assumed to be elements of the flow net and of uniform resistance per unit of length.

This suggests extension of the method to a general method of studying flow nets in moving water.

The methods apparently have important applications in the study of any field of potential.

Clearly many variations of technique are possible in applying the methods. Thus, a circuit is unbalanced only by counterflow in adjacent circuits, and so it is possible to merely "carry over" a certain fraction of such counterflows. Also it is possible to use the results from balancing other circuits in computing each successive circuit. Considerable experimenting with various procedures leads the writer to believe that the technique recommended will be found most satisfactory because of its simplicity.

It will bear repetition that the first approximations need not be made very formally. With some experience it is possible to nearly adjust a network at once by a little judicious guessing.

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