LOCALIZED TOPOLOGY CONTROL IN WIRELESS NETWORKS

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Abstract

Topology control has crucial impact on the system performance of wireless ad hoc networks. We propose several topology control algorithms that can maintain network connectivity while reducing energy consumption and improving network capacity. Being fully localized, these algorithms adapt well to mobility, and incur less overhead and delay. They not only significantly outperform existing schemes in terms of energy efficiency and network capacity, but also provide performance guarantees such as degree bound and min-max optimality.

We first present *Local Minimum Spanning Tree* (LMST) for homogeneous wireless ad hoc networks where each node has the same maximal transmission power, and prove several desirable properties. Then we show that most existing algorithms cannot be directly applied to heterogeneous networks where nodes have different maximal transmission power, and propose *Directed Relative Neighborhood Graph* (DRNG) and *Directed Local Spanning Subgraph* (DLSS). To the best of our knowledge, this is one the first efforts to address the connectivity and bi-directionality issues in heterogeneous wireless networks. We prove that the out-degree of any node in the resulting topology by LMST, DLSS or DRNG is upper-bounded by a constant. To incorporate fault tolerance into network topologies, we propose *Fault-tolerant Local Spanning Subgraph* (FLSS), which preserves k-vertex connectivity and is min-max optimal (i.e., the maximal transmission power among all nodes in the network is minimized) among all strictly localized algorithms.

We also examine several widely used assumptions in topology control (e.g., obstacle-free communication channel, the capability of obtaining position information), and discuss how to relax these assumptions to make our algorithms more practical.

Finally, we consider power-efficient broadcast in wireless ad hoc networks as an application of the proposed topology control algorithms, and propose *Broadcast on Local Spanning Subgraph* (BLSS), which broadcasts in a constrained flooding fashion over the network topology by FLSS. We show that BLSS is scalable, power-efficient, reliable, and significantly outperforms existing localized broadcast algorithms.

To my wife, Xiaofei, and my parents, for their love and support.

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Table of Contents

List of Figures

Chapter 1

Introduction

A wireless ad hoc network consists of a group of autonomous wireless devices that communicate with each other over the shared wireless channels. These wireless nodes are either mobile (e.g., in wireless mobile ad hoc networks) or static (e.g., in wireless sensor networks). Wireless ad hoc networks can be established very quickly since they do not require the support of a fixed infrastructure. Typical applications of wireless ad hoc networks include disaster recovery, surveillance, environment monitoring, just to name a few.

Wireless ad hoc networks are different from wired networks, wireless cellular networks [83], or wireless local area networks (WLANs) in that the topology may change constantly. Communication links are formed on the fly according to the distribution and mobility of nodes, and are dependent on the status of other links due to wireless interference in the physical and the MAC layers. This characteristic gives rise to several interesting phenomena. For one thing, network connectivity becomes a function of both the spatial distribution of nodes and the transmission power of each individual node. For another, the network capacity is determined not only by the connectivity, but also by MAC and routing protocols. As a consequence, several new challenges have emerged in the system design and analysis of wireless ad hoc networks, including:

- Energy efficiency [46, 51]. Since many wireless devices, especially mobile devices and sensors, are battery-powered, how to carry out the necessary functions with the minimal energy is critical to prolong the network lifetime.

- Network capacity [37]. The capacity of wireless ad hoc networks is limited in the sense that the wireless channels has to be shared among all devices that are within the transmission or carrier sensing range of each other. How to determine the transmission power and coordinate transmissions among devices so as to increase spatial reuse and maximize the network capacity has thus become an important issue.

- Channel access. How to arbitrate channel access among devices is crucial to enable the efficient sharing of the wireless channel and to provide certain level of performance guarantee (e.g., for real-time applications).
- Routing. This issue is concerned with the efficient and correct delivery of messages, given the dynamic network topology and the unreliable communication channel.
- Security and privacy. Since wireless communication is broadcast in nature, wireless traffic is subject to malicious attacks and privacy violation. How to ensure security and preserve privacy is a key issue to the wide deployment of wireless ad hoc networks.

Among the above challenges, energy efficiency and network capacity are the most fundamental to the network performance, as energy and bandwidth are the two major wireless resources. In this thesis, we aim to achieve energy efficiency and improve network capacity with topology control.

1.1 Topology Control

In a wireless network where every node transmits with its maximal transmission power, the network topology is built implicitly by the routing protocol (which update the routing cache continuously) [78] without considering the power issue. In particular, each node keeps a list of neighbor nodes that are within its transmission range. The key idea of topology control is that, instead of transmitting with the maximal power, nodes collaboratively determine the transmission power and define the network topology by forming the proper neighbor relation.

The importance of topology control lies in the fact that it critically affects the system performance in several ways. As shown in [37], it determines the network spatial reuse and hence the traffic carrying capacity. Choosing too large a power level results in excessive interference, while choosing too small a power level may result in a disconnected network. Topology control also affects the energy usage of communication, and thus impacts on the battery life. In addition, topology control also affects the contention for the medium. Contention and potential collision can be mitigated as much as possible by choosing the smallest transmission power subject to maintaining network connectivity [81] [99].

An effective topology control algorithm should meet several requirements. First, the algorithm should preserve network connectivity (or k-connectivity for the purpose of fault tolerance) by using minimal power. This is the most important objectives of topology control. Second, the algorithm should be distributed. Since there is usually no central authority in a multi-hop wireless network, each node has to make its own decision based on the information collected from the network. Third, the algorithm should depend only on the information collected *locally* so as to be less susceptible to mobility. Algorithms that only depend on local information also incur less message overhead as well as smaller communication delay. Finally, the topology derived under the algorithm should contain only bi-directional links, as bi-directional links guarantee the existence of reverse paths, and facilitate link-level acknowledgment [81] and proper operation of the channel reservation mechanisms such as RTS/CTS in IEEE 802.11.

1.2 Contributions

We propose several localized topology control algorithms that maintain network connectivity while reducing energy consumption and improving network capacity. These algorithms not only significantly outperform existing approaches in terms of energy efficiency, network capacity, and several other performance metrics, but also provide certain performance guarantees such as the degree bound and min-max optimality.

For homogeneous wireless multi-hop networks with uniform transmission ranges (which correspond to the maximal transmission power), we propose a fully localized topology control algorithm, *Local Minimum Spanning Tree* (LMST) [71, 72]. The network topology is constructed by each node independently building its local minimum spanning tree (with the information locally collected) and keeping only immediate on-tree nodes as neighbors. We prove that (1) the topology generated by LMST preserves the network connectivity; (2) the out-degree of any node in the resulting topology is upper-bounded by six; and (3) the resulting topology can be converted into one with only bi-directional links and network connectivity is still preserved. LMST is simple and scalable, and outperforms several existing algorithms in terms of network capacity, energy efficiency, and end-to-end delay.

For heterogeneous networks where nodes have nonuniform transmission ranges, we show that most existing algorithms cannot be directly applied. We then propose two localized topology control algorithms for heterogeneous networks: *Directed Relative Neighborhood Graph* (DRNG) and *Directed Local Spanning Subgraph* (DLSS) [69]. We prove that (1) the topology generated by DRNG or DLSS preserves strong connectivity of the network; (2) the out-degree of any node in the resulting topology by DLSS or DRNG is upper-bounded by a constant; and (3) the topology generated by DRNG or DLSS preserves network bi-directionality. To the best of our knowledge, this is one of the first efforts to address the connectivity and bi-directionality issues in heterogeneous wireless networks.

In spite of the many advantages, topology control algorithms usually decrease the number of links in the network, which reduces the number of possible routing paths. The topology derived is thus more susceptible to software/hardware failures. To address the fault tolerance issue, we first propose a centralized algorithm, *Fault-tolerant Global Spanning Subgraph* (FGSS_k) [65], that preserves k-vertex connectivity and is min-max optimal (i.e., the maximal transmission power among all nodes in the network is minimized). Based on the centralized algorithm, we then propose a fully localized algorithm, *Fault-tolerant Local Spanning Subgraph* (FLSS_k). We prove that FLSS_k preserves k-vertex connectivity and maintains bi-directionality for all the links in the topology, and is min-max optimal among all strictly localized algorithms. We further examine several widely used assumptions in topology control (e.g., a common maximal transmission power for all the nodes, obstacle-free communication channel, and capability of obtaining position information) and discuss how to relax these assumptions.

As broadcast is another important data dissemination mechanism in wireless ad hoc networks, we also consider the problem of power-efficient broadcast in wireless ad hoc networks as an application of our proposed topology control algorithms. We first show that *multi-hop* broadcast is usually more power-efficient. Then we propose *Broadcast on Local Spanning Subgraph* (BLSS) [64, 70]. An underlying topology is first constructed by FLSS_k, where the value of k determines the level of fault tolerance of the topology. Broadcast messages are then simply relayed through the derived topology in a constrained flooding fashion. BLSS is fully localized, scalable, power-efficient, and fault-tolerant: (1) it does not rely on any specific power consumption model; (2) it requires only local (rather than global) information and adapts well to mobility; and (3) it provides a reliable shared broadcast infrastructure. Simulation results show that BLSS significantly outperforms existing localized algorithms, and is comparable to existing centralized algorithms.

1.3 Thesis Outline

The rest of this thesis is organized as follows. We first provide a literature review on topology control and related issues, as well as a description of our network model in Chapter 2. Then we present the MST-based topology control algorithm, LMST, in Chapter 3 for homogeneous wireless network with uniform transmission ranges. Following that, we discuss two localized topology control algorithms, DRNG and DLSS, in Chapter 4 for heterogeneous wireless networks with non-uniform transmission ranges. To address the fault tolerance issue in wireless networks, we propose a centralized algorithm, FGSS, and a localized algorithm, FLSS, in in Chapter 5. Then we propose BLSS, a scalable and power-efficient broadcast algorithm for wireless ad hoc networks, in Chapter 6. Finally, we conclude the thesis by recapitulating the contributions and giving research avenues for future work in Chapter 7.

1.4 Publication Notes

Part of the results in Chapter 3 were published in the proceedings of the *22nd Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM 2003)* [71] and in *IEEE Transactions on Wireless Communications*, vol. 4, no. 3, 2005 [72]. Part of the results in Chapter 4 were published in the proceedings of the *23rd Annual Conference of the IEEE Communication Society (INFOCOM 2004)* [66] and are to appear in *IEEE/ACM Transactions on Networking* [69]. Part of the results in Chapter 5 were published in the proceedings of the *Tenth Annual International Conference on Mobile Computing and Networking (MOBICOM 2004)* [65] and are to appear in *IEEE Transactions on Parallel and Distributed Systems* [68]. Part of the results in Chapter 6 were published in the proceedings of the *1st International Conference on Quality of Service in Heterogeneous Wired/Wireless Networks (QSHINE 2004)* [64] and are to appear in *ACM/Baltzer Wireless Networks* [70]. Some results in this thesis also appeared (or are to appear) in:

- R. Zheng, J. C. Hou, and N. Li. Power management and control in wireless networks. In Y. Pan and Y. Xiao, editors, *Ad Hoc and Sensor Networks: Wireless Networks and Mobile Computing, Volume 2*. Nova Science Publishers, 2005 [123].

- J. C. Hou, N. Li, and I. Stojmenović. Topology construction and maintenance in wireless sensor networks. In I. Stojmenović, editor, *Handbook of Sensor Networks: Algorithms and Architecture*. John Wiley and Sons, 2005 [43].

- N. Li and J. C. Hou. Topology control in wireless networks. In D.-Z. Du, M. Cheng, and Y. Li, editors, *Combinatorial Optimization in Communication Networks*. Kluwer Academic Publishers, in press [67].

Chapter 2

Preliminaries

In this chapter, we first summarize the state of the art in the research related to maintaining network connectivity and deriving network topologies. Then we define the network model to be used throughout this thesis.

2.1 Literature Review

Connectivity, capacity and topology control are closely related issues in wireless ad hoc networks. In this section, we will review related work in these three areas.

2.1.1 Network Connectivity

Network connectivity has always been a research focus in wireless ad hoc networks. It is an indispensable function for network services to be in place. Few network services can operate effectively if the network is partitioned.

Gupta and Kumar [36] studied the critical common range r_n for the connectivity of n independently and uniformly distributed wireless nodes in a disk of unit area. Their analysis showed that the network is asymptotically connected with probability 1 as the number of nodes in the network goes to infinity if and only if $c(n) \to \infty$, where r_n satisfies that $\pi r_n^2 = \frac{\log n + c(n)}{n}$.

In an independent effort, Penrose showed in [87] that M_n, the length of the longest edge in the minimum spanning tree of n points randomly and uniformly distributed in a unit area square, satisfies that $\lim_{n\to\infty} \Pr(n\pi M_n^2 - \ln n \leq \alpha) = e^{-e^{-\alpha}}$. This result is strong than that in [36] in the sense that it gives the exact expression of the probability of the connectivity. Sánchez *et al.* [98] also used a similar approach to find the critical transmission range.

From a different perspective, Xue and Kumar [120] studied the relationship between connectivity and node degree. They assumed the same number of nearest neighbors are maintained for each node, and showed that (i) the network is asymptotically disconnected with probability 1 as n goes to infinity, if each node is connected to less than $0.074 \log n$ nearest neighbors; and (ii) the network is asymptotically connected with probability 1 as n goes to infinity, if each node is connected to more than $5.1774 \log n$ nearest neighbors. Wan and Yi [113] further studied the critical number of neighbors for maintaining k-connectivity and found the upper bound to be $\alpha e \log n$, where $\alpha > 1$ is a real number and $e \simeq 2.718$ is the natural base.

Dousse *et al.* [28] considered the connectivity of both pure ad hoc and hybrid large-scale wireless networks, where the density of nodes is low and the interference is less critical. The results showed that introducing a sparse network of base stations can greatly improve the connectivity in one dimensional cases, but does not change the connectivity of 2-dimensional networks significantly.

The network connectivity in the presence of interference is studied in [26] by Dousse *et al.* In their model, two nodes can communicate with each other if the signal to noise ratio at the receiver is larger than a threshold, where the noise is composed of the sum of interferences from all other transmitters, weighted by a coefficient γ, and a background white noise. They found that there exists a critical value of γ, above which the network is partitioned into isolated clusters.

2.1.2 Network Capacity

Network capacity, a key characteristic of wireless networks, can be defined as the long-term achievable throughput rate per node with high probability. Unlike in wired networks where throughput capacity can be calculated straightforwardly using standard techniques, the capacity of a wireless network depends on several factors such as node deployment, traffic distribution, PHY/MAC characteristics, and routing strategies.

In their groundbreaking work [37], Gupta and Kumar studied the capacity of static wireless networks. For a *Random Network* of n identically and independently randomly distributed nodes, the throughput $\lambda(n)$ obtainable by each node for a randomly chosen destination is $\Theta(\frac{W}{\sqrt{n \log n}})$ bits per second under a non-interference protocol, where W is the channel capacity. For an *Arbitrary Network* where the nodes are optimally placed in a disk of unit area, traffic patterns are optimally assigned, and each transmission's range is optimally chosen, the bit-distance product that can be transported over the entire network is $\Theta(W\sqrt{n})$

bit-meters per second.

Toumpis and Goldsmith [109] extended the results of [37] to a 3-dimensional networks, and incorporated the Shannon capacity into the link model. It is found that the capacity $C(n)$ of a wireless network satisfies the inequality $k_1 \frac{n^{1/3}}{\log n} \leq C(n) \leq k_2 \log n \cdot n^{1/2}$.

Li *et al.* [61] examined the interaction of 802.11 MAC and ad hoc forwarding, and their effect on capacity for several configurations and traffic patterns. They showed that for the total capacity to scale up with the network size, the average distance between the source and the destination nodes must remain small as the network grows in size.

Grossglauser and Tse [35] investigated the capacity of mobile ad hoc wireless networks. Under the model where n nodes communicate in random source-destination pairs and each node has unlimited buffer, the per-session throughput for application with loose delay requirement can reach $\Theta(1)$ when nodes are mobile. With mobility, the per-session throughput increases dramatically compared with that in [37]. However, the delay can be unbounded due to the unlimited buffering of packets in the relaying nodes.

Bansal and Liu [5] also considered the capacity of mobile ad hoc wireless networks. They proposed a routing algorithm which exploits the mobility patterns of nodes to provide the guarantee on the delay. The throughput achieved by the algorithm is only a poly-logarithmic factor off from the optimal results in [35].

Liu *et al.* [77] studied the throughput capacity of a hybrid wireless network, which is an ad hoc wireless network with a sparse high-bandwidth wired network of base stations acting as relays. Two different routing strategies are considered. In the first strategy, data are forwarded through the infrastructure if the destination is outside of the cell where the source is located; otherwise, the data are forwarded in a multi-hop fashion as in an ad hoc network. In the second strategy, a node chooses whether to use the infrastructure to send data with probability p. For a hybrid network of n nodes and m base stations, it is shown that if m grows slower than $\sqrt{n}$ asymptotically, the benefit of adding base stations on capacity is insignificant. On the other hand, if m grows faster than $\sqrt{n}$, the throughput capacity increases linearly with the number of base stations.

Kozat and Tassiulas [56] assumed a different hybrid network model and concluded that the per source node capacity of $\Theta(W/\log(N))$ can be achieved, if the number of ad hoc nodes per access points is bounded above. They also found that a per source node capacity figure that is arbitrarily close to $\Theta(1)$ can not be obtained in this model.

Yi *et al.* extended the work in [37] to wireless nodes using directional antennas. The pattern of directional antennas is approximated as a circular sector with radius r and angle α or β depending on the mode of the antenna (α for transmitter and β for receiver). They showed that for Arbitrary Networks, the capacity gain is $\sqrt{\frac{2\pi}{\alpha}}$, $\sqrt{\frac{2\pi}{\beta}}$ and $\frac{2\pi}{\sqrt{\alpha\beta}}$ when using directional transmission with omni reception, omni transmission with directional reception, and directional transmission with directional reception, respectively. For Random Networks, the capacity gain is $\frac{2\pi}{\alpha}$,$\frac{2\pi}{\beta}$, and $\frac{4\pi^2}{\alpha\beta}$, respectively. These results show that directional antennas can only provide a constant factor of gain.

Peraki and Servetto [122] formulated the problem of maximizing the stable throughput as one that finds maximum flows on random unit-disk graphs. They found that an increase of $\Theta(\log^2 n)$ in the maximum stable throughput is all that can be achieved by allowing arbitrarily complex signal processing at the transmitters and receivers. Therefore, neither directional antennas nor the ability to communicate simultaneously with multiple nodes can effectively improve the capacity of wireless networks in practice. The major pitfall of this work is that the formulation of the problem does not reflect the fact that the wireless channel is shared. Moreover, it is almost impossible to find the minimum cut of a wireless network without the exact information on network traffic.

Dousse and Thiran [27] investigated the trade-off between the network capacity and connectivity. The analysis shows that the attenuation function has significant impact on the network properties. For commonly used power law attenuation functions, the attainable rate per node scales as $1/\sqrt{n}$ and connectivity can be ensured. For attenuation functions that is uniformly bounded and does not have a singularity at the origin, the attainable rate per node has to scale as $1/n$ to ensure connectivity.

Gamal *et al.* [33] studied the fundamental trade-off between capacity and delay in wireless networks. Their analysis showed that, for the Gupta-Kumar static network model [37], the optimal throughput-delay trade-off is given by $D(n) = \Theta(nT(n))$, where $D(n)$ is the average delay and $T(n)$ is the achievable throughput per node. For the Grossglauser-Tse mobile network model [35], the delay scales as $\Theta(\frac{\sqrt{n}}{v(n)})$, where $v(n)$ is the velocity of the mobile nodes. They also proposed schemes to achieves the optimal order of delay for any given throughput in a mobile network.

2.1.3 Topology Control

Topology control in wireless ad hoc networks has been proved to be NP-hard under different settings [8, 20–22, 55]. As a result, various heuristics have been proposed.

Rodoplu and Meng [96] (referred as R&M thereafter) introduced the notion of *relay region* and *enclosure* for the purpose of power control. For any node i that attempts to transmit to node j, node j is said to be in the *relay region* of a third node r, if node i will consume less power when it chooses to relay through node r. The *enclosure* of node i is defined as the union of the complement of relay regions of all the nodes that node i can reach by using its maximal transmission power. A two-phase distributed protocol was then used to find the minimum power topology for a static network. In the first phase, each node i builds the enclosure graph. In the second phase, each node executes the distributed Bellman-Ford shortest path algorithm. It is shown that the network is strongly connected if every node maintains links with the nodes in its enclosure and the resulting topology is a minimum power topology. This algorithm assumes that there is only one data sink (destination) in the network, which may not hold in practice. Also, an explicit propagation channel model is needed to compute the relay region.

Based on the work in [96], Li and Halpern [62] proposed SMECN (Small Minimum-Energy Communication Network) to compute the minimum-energy topology in a more efficient way. The resulting network is generally a subnetwork of that computed by the algorithm in [96]. Li and Wan [74] also proposed an efficient distributed algorithm to construct an approximation of the minimum-energy topology in [96].

Ramanathan and Rosales-Hain [94] presented two centralized algorithms, CONNECT and BICONN-AUGMENT, to minimize the maximal power used per node while maintaining the (bi)connectivity of the network. Both are simple greedy algorithms that iteratively merge different components until only one remains. They also introduced two distributed heuristics for mobile networks. In LINT, each node is configured with three parameters - the "desired" node degree d_d, a high threshold d_h on the node degree, and a low threshold d_l. Every node will periodically check the number of active neighbors and change its power level accordingly, so that the node degree is kept within the thresholds. LILT further improves LINT by overriding the high threshold when the topology change indicated by the routing update results in undesirable connectivity. Both centralized algorithms require global information, and thus cannot be directly deployed in the case of mobility. Meanwhile, the two distributed heuristics may not preserve network connectivity.

COMPOW [81] and CLUSTERPOW [53] are approaches implemented in the network layer. Both hinge on the idea that if each node uses the smallest common power required to maintain network connectivity, the traffic carrying capacity of the entire network is maximized, the battery life is extended, and the MAC-level contention is mitigated. In COMPOW, each node runs several routing daemons in parallel, one for

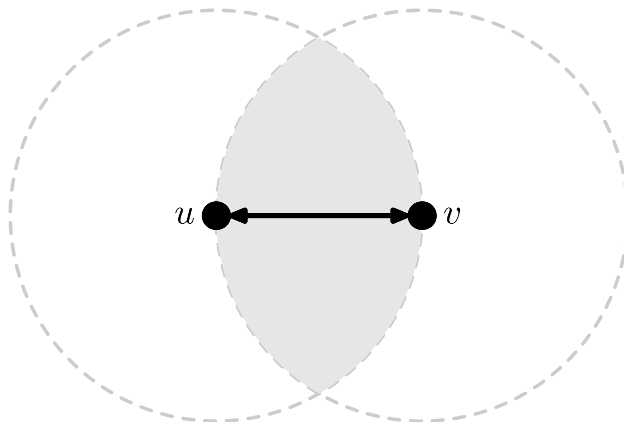


Figure 2.1: Definition of the Relative Neighborhood Graph (RNG)

each power level. Each routing daemon maintains its own routing table by exchanging control messages at the specified power level. By comparing the entries in different routing tables, each node can determine the smallest common power that ensures the maximal number of nodes are connected. CLUSTERPOW further deals with non-uniform distribution of nodes and provides adaptive and distributed clustering based on transmission power. The major drawback of these two approaches is their significant message overhead, since every node has to run multiple routing daemons, each of which has to exchange link state information with the counterparts at other nodes.

In CBTC(α) [63], every node increases its transmission power until either the maximum angle between any two consecutive neighbors is at most α or the maximal power is reached. The algorithm has been proved to preserve network connectivity if $\alpha \leq 5\pi/6$. Several optimization methods are also discussed to further reduce the transmission power after the topology is derived under the base algorithm. An event-driven strategy is proposed to reconfigure the network topology in the case of mobility, where a node is notified when any neighbor leaves/joins the neighborhood and/or the angle changes, and determines whether it needs to rerun the topology control algorithm.

Proximity graphs, including the Gabriel Graph (GG) [32,48], the *Relative Neighborhood Graph* (RNG) [107,110] and the Delaunay Triangulation (DT) [25], have also been proposed for topology construction [93] in wireless networks. For a given undirected graph G, $RNG(G) \subseteq GG(G) \subseteq DT(G)$ [43]. An edge with two end-vertices u and v is in the RNG if and only if there does not exist a third node p such that $d(u,p) < d(u,v)$ and $d(p,v) < d(u,v)$, where $d(\cdot,\cdot)$ is the Euclidean distance. Equivalently, $uv \in RNG$ if there exists no node inside the shaded area as shown in Figure 2.1.

Borbash and Jennings [12] proposed to use RNG for topology initialization of wireless networks. Based on the local knowledge, each node makes decisions to derive the network topology based on RNG. The derived network topology is shown to exhibit good performance in terms of power usage, interference, and reliability. The XTC algorithm proposed by Wattenhofer and Zollinger [114] is also based on RNG.

Li *et al.* [73] presented the Localized Delaunay Triangulation, a localized protocol that constructs a planar spanner of the *Unit Disk Graph* (UDG). The topology contains all edges that are both in the unit-disk graph and the Delaunay triangulation of all nodes. It is proved that the shortest path in this topology between any two nodes u and v is at most a constant factor of the shortest path connecting u and v in UDG. However, the notion of UDG and Delaunay triangulation cannot be directly extended to heterogeneous networks where different nodes may have different maximal transmission power..

Topology control algorithms do not always efficiently reduce the communication interference. Burkhart *et al.* [14] studied the problem of creating low-interference topology. They proposed a centralized algorithm, *Low Interference Forest Establisher* (LIFE), that computes a minimal-interference topology, and a distributed algorithm, *Local Low Interference Spanner Establisher* (LLISE), that computes a interference-optimal spanner.

Instead of adjusting the transmission power of individual devices, there also exist other approaches to generate power-efficient network topologies. By following a probabilistic approach, Santi *et al.* derived the suitable common transmission range that preserves network connectivity, and established the lower and upper bounds on the probability of connectedness [99]. In [6], a "backbone protocol" is proposed to manage large wireless ad hoc networks, in which a small subset of nodes is selected to construct the backbone. In [119], a method of calculating the power-aware connected dominating sets was proposed to establish an underlying topology for the network.

2.1.4 K-vertex Connectivity

k-vertex connectivity is important in topology control when fault tolerance has to be taken into account. A graph G is k-vertex connected if for any two vertices v_1, v_2 in G, there are at least k pairwise-vertex-disjoint paths from v_1 to v_2. Or equivalently, a graph is k-vertex connected if the removal of any $k-1$ nodes (and all the related links) does not partition the network. In wireless networks, we are more concerned with k-vertex connectivity (rather than k-edge connectivity) since the failure of at most $k-1$ nodes will not disconnect the network. We henceforth use k-connectivity to refer to k-vertex connectivity for notational simplicity.

The problem of finding a minimum-cost, k-connected subgraph is proved to be NP-hard [54]. Many approximation algorithms have been proposed (see [40] and [54] for a summary). Although most topology control algorithms do not take fault tolerance into consideration, there are several research efforts on studying the properties of k-connected topologies [7,88], devising algorithms to construct such topologies [4,40], or both [75].

Penrose [88] studied k-connectivity in a geometric random graph of n nodes derived by adding an edge between each pair of nodes that are at most r apart. He proved that the minimum value of r at which the graph is k-connected is equal to the minimum value of r at which the graph has the minimum degree of k, with probability 1 as n goes to infinity. The significance of this result is that it links k-connectivity, a global property of the graph, to node degree, a local parameter. It is hence possible to come up with localized algorithms that can preserve asympototic k-connectivity. However,the minimum value of r is not given in this work.

Bettstetter [7] also investigated the relation between the minimum node degree and k-connectivity for geometric random graphs. The analytical expression of the required range r_0 for the almost surely k-connected network is derived, and then verified by simulations.

Li *et al.* [75] extended Penrose's work and gave the lower bound and the upper bound on the minimum value of r at which the graph is k-connected with a high probability. The analysis shows that, for a unit-area square region, the probability that the network of n nodes is k-connected is at least $e^{-e^{-\alpha}}$, if the common transmission radius r_n satisfies $\pi r_n^2 \geq \ln n + (2k-3)\ln\ln n - 2\ln(k-1)! + 2\alpha$, for $k > 0$ and sufficiently large n, where α is any real number. Under the *homogeneous* network assumption, they also proposed a localized topology control algorithm that preserves k-connectivity. The proposed structure, Yao$_{p,k}$, is based on the Yao structure, and is constructed by having every node u choose k closest neighbors in each of the $p \geq 6$ equal cones around u. Yao$_{p,k}$ is proved to preserve k-connectivity and is a length spanner. It is not clear whether or not, and how, the proposed algorithm can be extended to accommodate *heterogeneous* networks.

Bahramgiri *et al.* [4] augmented the CBTC algorithm [63] to provide fault tolerance. Specifically, let $D(\alpha)$, the directed subgraph of G, be the output of CBTC(α) algorithm, and let $G(\alpha)$ be the result of applying *Removal* on $D(\alpha)$. It is proved that $G(\frac{2\pi}{3k})$ preserves the k-connectivity of G. As the work is extended from the CBTC algorithm, it shares the same assumption of a homogeneous network, which may not always hold in practice [69].

Hajiaghayi *et al.* [40] presented three approximation algorithms to find the minimum power k-connected subgraph. Two global algorithms are based on existing approaches. The first algorithm gives an $O(k\alpha)$-approximation, where α is the best approximation factor for the k-UPVCS problem defined in the paper. The second algorithm improves the approximation factor to $O(k)$ for general graphs. The third algorithm, *Distributed k-UPVCS*, is a distributed algorithm that gives a $k^{O(c)}$-approximation, where c is the exponent in the propagation model. For 2-connectivity, it first computes the minimum spanning tree (MST) of the input graph by using a distributed algorithm, and then adds an arbitrary path amongst the neighbors of each node in the returned tree. Since this distributed algorithm is based on the distributed MST algorithm, it is not fully localized, i.e., it relies on information that is multiple hops away to construct the MST. This implies more maintenance overhead and delay will be incurred when the topology has to be adjusted in response to node mobility or failure. Moreover, a closer investigation of the distributed algorithm reveals that the neighbors of a node on the minimum spanning tree may not be able to communicate with each other due to the limited transmission power. As a result, the "arbitrary path connecting neighbors" in the algorithm (see [40]) may not exist in a network of low density. A counter-example can be found in [65].

2.2 Network Model

In this section, we define the network model to be used throughout this thesis. Let the topology of a multi-hop wireless ad hoc network be represented by a simple directed graph $G = (V(G), E(G))$ in the 2-D plane, where $V(G) = \{v_1, v_2, \ldots, v_n\}$ is the set of nodes (or equivalently, vertices) and $E(G)$ is the set of links (or equivalently, edges) in the network. Each node has a maximal transmission power, which corresponds to its transmission range. Each node is assigned a unique identifier, *id* (such as an IP/MAC address).

Although G is usually assumed to be geometric in the literature, here we only require that G is a general graph, i.e., $E(G) = \{(u, v) : v \text{ can receive } u\text{'s transmission correctly } , u, v \in V(G)\}$. We also assume that the wireless channel is symmetric (i.e., both the sender and the receiver should observe the same channel properties such as path loss and fading), and each node is able to gather its own location information via, for example, several lightweight localization techniques for wireless networks [39, 41, 44, 84, 102].

Definition 2.1 (Visible Neighborhood). *The* visible neighborhood N_u^V *is the set of nodes that node* u *can reach by using its maximal transmission power, i.e.,* $N_u^V = \{v \in V(G) : (u,v) \in E(G)\}$. *For each node* $u \in V(G)$*, let* $G_u^V = (V(G_u^V), E(G_u^V))$ *be the induced subgraph of* G *such that* $V(G_u^V) = N_u^V$ *and* $E(G_u^V) = \{(u,v) \in E(G) : u, v \in V(G_u^V)\}$.

Each edge in $E(G)$ is assigned a weight. Two points are worth mentioning here. First, to build a power efficient spanning subgraph, the weight of an edge is usually the power consumption of a transmission between the two end-vertices. For the algorithms to be presented later, it suffices to use the Euclidean distance as the weight as the weight function. The resulting topology will be the same, since the power consumption is, in general, of the form $c_0 \cdot d^\alpha + c_1$, $\alpha \geq 2$, which is a strictly increasing function of the Euclidean distance. More discussions will be given in Section 5.3.4 on how to use the power consumption as the measure of weight. Second, to ensure that two edges with different end-vertices have different weights, we use the *id*s of the end-vertices as tie-breakers.

Definition 2.2 (Weight Function). *For an edge* $e = (u,v)$*, the* weight function $w : E \mapsto R^3$ *maps to a 3-tuple, i.e.,* $w(u,v) = (d(u,v), \max\{id(u), id(v)\}, \min\{id(u), id(v)\})$. *Given* $(u_1,v_1),(u_2,v_2) \in E$,

$$\begin{aligned} w(u_1,v_1) > w(u_2,v_2) \quad \Leftrightarrow \quad & d(u_1,v_1) > d(u_2,v_2) \\ \textit{or} \quad & (d(u_1,v_1) = d(u_2,v_2) \;\&\&\; \max\{id(u_1), id(v_1)\} > \max\{id(u_2), id(v_2)\}) \\ \textit{or} \quad & (d(u_1,v_1) = d(u_2,v_2) \;\&\&\; \max\{id(u_1), id(v_1)\} = \max\{id(u_2), id(v_2)\} \\ & \&\& \; \min\{id(u_1), id(v_1)\} > \min\{id(u_2), id(v_2)\}). \end{aligned}$$

It is obvious that two edges with different end-vertices have different weights, and two edges with the same end-vertices have the same weight, i.e., $w(u,v) = w(v,u)$.

Definition 2.3 (Neighbor Set). *Node* v *is an* out-neighbor *of node* u *(and* u *is an* in-neighbor *of* v*) under an algorithm* ALG*, denoted* $u \xrightarrow{ALG} v$*, if and only if there exists an edge* (u,v) *in the topology generated by the algorithm. In particular, we use* $u \rightarrow v$ *to denote the neighbor relation in* G. $u \xleftrightarrow{ALG} v$ *if and only if* $u \xrightarrow{ALG} v$ *and* $v \xrightarrow{ALG} u$. *The* out-neighbor set *of node* u *is* $N_{ALG}^{out}(u) = \{v \in V(G) : u \xrightarrow{ALG} v\}$*, and the* in-neighbor set *of* u *is* $N_{ALG}^{in}(u) = \{v \in V(G) : v \xrightarrow{ALG} u\}$.

Definition 2.4 (Radius). R_u*, the* radius *of node* u *is defined as the Euclidean distance between* u *and its farthest neighbor, i.e,* $R_u = \max_{v \in N_{ALG}^{out}(u)}\{d(u,v)\}$.

Definition 2.5 (Degree). *The* out-degree *of a node u under an algorithm ALG, denoted $deg^{out}_{ALG}(u)$, is the number of out-neighbors of u, i.e., $deg^{out}_{ALG}(u) = |N^{out}_{ALG}(u)|$. Similarly, the* in-degree *of a node u, denoted $deg^{in}_{ALG}(u)$, is the number of in-neighbors, i.e., $deg^{in}_{ALG}(u) = |N^{in}_{ALG}(u)|$.*

Note that the degree defined above is also referred as the *logical* node degree. It is often necessary to consider the *physical* node degree, i.e., the number of nodes within the transmission radius.

Definition 2.6 (Topology). *The* topology *generated by an algorithm ALG is a directed graph $G_{ALG} = (V(G_{ALG}), E(G_{ALG}))$, where $V(G_{ALG}) = V(G)$, and $E(G_{ALG}) = \{(u, v) \in E(G) : u \xrightarrow{ALG} v\}$.*

Definition 2.7 (Connectivity). *For any topology generated by an algorithm ALG, node u is said to be* connected to *node v (denoted $u \Rightarrow v$) if there exists a path $(p_0 = u, p_1, \ldots, p_{m-1}, p_m = v)$ such that $p_i \xrightarrow{ALG} p_{i+1}, i = 0, 1, \ldots, m-1$, where $p_k \in V(G_{ALG}), k = 0, 1, \ldots, m$. It follows that $u \Rightarrow v$ if $u \Rightarrow p$ and $p \Rightarrow v$ for some $p \in V(G_{ALG})$.*

Definition 2.8 (Bi-Directionality). *A topology generated by an algorithm ALG is* bi-directional, *if for any two nodes $u, v \in V(G_{ALG})$, $v \in N^{out}_{ALG}(u)$ implies $u \in N^{out}_{ALG}(v)$.*

Definition 2.9 (Bi-Directional Connectivity). *For any topology generated by an algorithm ALG, node u is said to be* bi-directionally connected to *node v (denoted $u \Leftrightarrow v$) if there exists a path $(p_0 = u, p_1, \ldots, p_{m-1}, p_m = v)$ such that $p_i \xleftrightarrow{ALG} p_{i+1}, i = 0, 1, \ldots, m-1$, where $p_k \in V(G_{ALG}), k = 0, 1, \ldots, m$. It follows that $u \Leftrightarrow v$ if $u \Leftrightarrow p$ and $p \Leftrightarrow v$ for some $p \in V(G_{ALG})$.*

Definition 2.10 (Addition and Removal). *The* Addition *operation is to add an extra edge (v, u) into G_{ALG} if $(u, v) \in E(G_{ALG})$ and $(v, u) \notin E(G_{ALG})$. The* Removal *operation is to delete any edge $(u, v) \in E(G_{ALG})$ if $(v, u) \notin E(G_{ALG})$.*

Both *Addition* and *Removal* operations attempt to create a bi-directional topology by either converting uni-directional edges into bi-directional ones or removing all uni-directional edges. The resulting topology after *Addition* or *Removal* is alway bi-directional, if the maximal transmission power for each node is the same. If the maximal transmission power for each node is not the same, the result of *Removal* is still bi-directional, while the result of *Addition* may not be bi-directional (see Chapter 4 for more detailed discussions).

Chapter 3

Localized Topology Control in Homogeneous Networks

In this chapter, we consider a homogeneous wireless network where every node has the same maximal transmission range d_{max}. This assumption is also referred as the *Unit Disk Graph* (UDG) assumption [19], which is widely used in the literature. As a result, the network topology G becomes an undirected graph. We now propose a localized topology control algorithm, *Local Minimum Spanning Tree* (LMST) [71], for homogeneous networks.

Several topology control algorithms [53,63,81,94,96] have been proposed to create a power-efficient network topology in wireless ad hoc networks with limited mobility, among which R&M [96], CBTC(α) [63], COMPOW [81] and CLUSTERPOW [53], and CONNECT [94] may have received the most attention. Some of the algorithms require explicit propagation channel models (e.g., R&M), while others incur significant message exchanges (e.g., COMPOW/CLUSTERPOW). Their ability to maintain the topology in the case of mobility is also rather limited.

We propose LMST, a Minimum Spanning Tree (MST) based topology control algorithm. The topology is constructed by having each node build its local MST independently and keep only immediate on-tree nodes as neighbors. There are several salient features of LMST: (1) the topology generated by LMST preserves the network connectivity, (2) the out-degree of any node in the resulting topology is bounded by 6; and (3) the resulting topology can be converted into one with only bi-directional links. Feature (2) is desirable because a small out-degree reduces the MAC-level contention and interference. The capability of forming a topology that consists of only bi-directional links is important for link level acknowledgments, and packet transmissions and retransmissions over the unreliable wireless medium. Bi-directional links are also important for floor acquisition mechanisms such as RTS/CTS in IEEE 802.11.

Simulation results indicate that compared with the other localized topology control algorithms, LMST has smaller average out-degree (both logical and physical) and smaller average radius. The former reduces the MAC-level contention, while the latter implies that only small transmission power is needed to maintain connectivity. LMST also outperforms the other algorithms in terms of the total amount of data delivered, the energy efficiency, and the end-to-end delay.

The rest of this chapter is organized as follows. We present the LMST algorithm in Section 3.1 and prove several of its desirable properties, i.e., preservation of network connectivity, bound on the out-degree, and bi-directionality, in Section 3.2. Moreover, the frequency to update the topology in case of limited mobility is determined using a probabilistic model in Section 3.2.3. Finally, we present the performance study in Section 3.3 and conclude this chapter in Section 3.4.

3.1 LMST: Local Minimum Spanning Tree

The proposed algorithm consists of three steps: information collection, topology construction, and construction of topology with only bi-directional links.

3.1.1 Information Collection

The information needed by each node u in the topology construction process is the information of all nodes in its visible neighborhood, N_u^V. This can be obtained by having each node broadcast periodically a Hello message using its maximal transmission power. The information contained in a Hello message should at least include the node *id* and the position of the node. Those periodic messages can be sent either in the data channel or in a separate control channel. The Hello messages can be combined with those already employed in most ad hoc routing protocols. In addition, each node can piggy-back its location information in data packets to reduce the number of Hello exchanges. The time interval between two broadcasts of Hello messages depends on the level of node mobility, and will be determined by the probabilistic model to be introduced in Section 3.2.3.

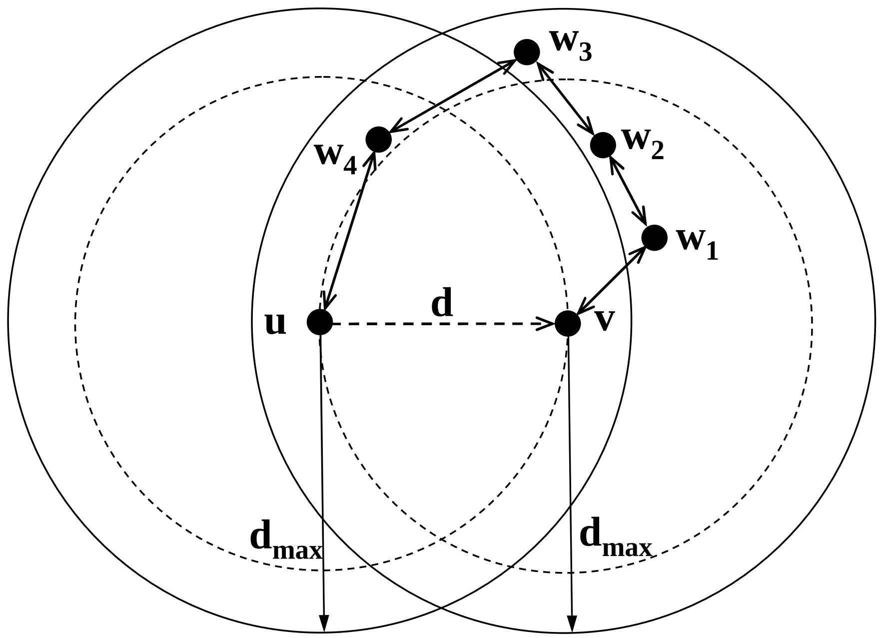


Figure 3.1: The topology by LMST may be uni-directional

3.1.2 Topology Construction

After obtaining the neighborhood information, each node u builds its local minimum spanning tree T_u that spans all the nodes within its visible neighborhood N_u^V. The time it takes to build the MST varies from $O(m \log n)$ (the original Prim's algorithm [91]) to almost linear of m (the optimal algorithm [90]), where n is the number of vertices and m is the number of edges. Node v is a neighbor of node u if and only if (u, v) is a link on the local MST built by u. The network topology under LMST is all the nodes in V and their individually perceived neighbor relations. Note that the topology is *not* a simple superposition of all local MSTs.

Definition 3.1 (LMST)**.** *In Local Minimum Spanning Tree (LMST), node v is a neighbor of node u, denoted $u \xrightarrow{LMST} v$, if and only if $(u, v) \in E(T_u)$. That is, v is a neighbor of u if and only if v is an immediate neighbor on u's local MST T_u.*

3.1.3 Construction of Topology with Only Bi-directional Links

Since the neighbor relation is decided locally by each node, some links in the final topology may be uni-directional, i.e., $u \xrightarrow{LMST} v$ does not necessarily imply $v \xrightarrow{LMST} u$. In Figure 3.1, $d(u, v) = d < d_{max}$, $d(u, w_4) < d_{max}$, $d(u, w_i) > d_{max}, i = 1, 2, 3$, and $d(v, w_j) < d_{max}, j = 1, 2, 3, 4$. Since $N_u^V =$

$\{u, v, w_4\}$, $u \xrightarrow{LMST} v$ and $u \xrightarrow{LMST} w_4$. Also $N_v^V = \{u, v, w_1, w_2, w_3, w_4\}$, hence $v \xrightarrow{LMST} w_1$. Here link (u, v) is uni-directional. We can apply either *Addition* or *Removal* to obtain a bi-directionally connected topology.

Definition 3.2 (Topology G^+_{ALG}). *The topology G^+_{ALG} generated by an algorithm ALG is an undirected graph $G^+_{ALG} = (V(G^+_{ALG}), E(G^+_{ALG}))$, where $V(G^+_{ALG}) = V(G_{ALG})$, and $E^+_{ALG} = \{(u, v) : (u, v) \in E(G_{ALG})$ or $(v, u) \in E(G_{ALG})\}$.*

Definition 3.3 (Topology G^-_{ALG}). *The topology G^-_{ALG} generated by an algorithm ALG is an undirected graph $G^-_{ALG} = (V(G^-_{ALG}), E(G^-_{ALG}))$, where $V(G^-_{ALG}) = V(G_{ALG})$, and $E^-_{ALG} = \{(u, v) : (u, v) \in E(G_{ALG})$ and $(v, u) \in E(G_{ALG})\}$.*

To convert G_{LMST} into either G^+_{LMST} or G^-_{LMST}, every node u may probe each of its neighbors in the out-neighbor set $N^{out}_{LMST}(u)$ to find out whether or not the corresponding edge is uni-directional. If any uni-directional edge is found, u will either delete the edge or notify its neighbor to add the reverse edge. In Section 3.2, we will prove that both new topologies preserve the desirable properties of G_{LMST}. There exists a trade-off between the two choices: the latter gives a comparatively simpler topology, and hence is more efficient in terms of spatial reuse, while the former allows more routing redundancy.

3.1.4 Determination of Transmission Power

Assume that the maximal transmission power is known and is the same to all nodes. By measuring the receiving power of Hello messages, each node can determine the specific power levels it needs to reach each of its neighbors. In what follows, we first describe two commonly-used propagation models [95], and then elaborate on how to determine the transmission power. Note that this approach can be applied to any radio propagation model.

In the Free Space propagation model, the relation between the power used to transmit packets, P_t and the power received, P_r can be characterized as

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2 L}, \tag{3.1}$$

where G_t is the antenna gain of the transmitter, G_r is the antenna gain of the receiver, λ is the wave length, d is the distance between the antenna of the transmitter and that of the receiver, and L is the system loss.

In the Two-Ray Ground propagation model, the relation between P_t and P_r is

$$P_r = \frac{P_t G_t G_r h_t^2 h_r^2}{d^4 L}, \tag{3.2}$$

where G_t is the antenna gain of the transmitter, G_r is the antenna gain of the receiver, h_t is the antenna height of the transmitter, h_r is the antenna height of the receiver, d is the distance between the antenna of the transmitter and that of the receiver, and L is the system loss.

In general, the relation between P_t and P_r is of the form

$$P_r = P_t \cdot G, \tag{3.3}$$

where G is a function of G_t, G_r, h_t, h_r, λ, d, α, L, and is time-invariant if some of the parameters are time-invariant. At the information collection stage, each node broadcasts its position using the maximal transmission power P_{max}. When node A receives the position information from node B, it measures the receiving power P_r and obtains $G = P_r/P_{max}$. Henceforth node A needs to transmit using at least $P_{th} \cdot G = P_{th}P_r/P_{max}$ so that node B can receive messages, where P_{th} is the power threshold to correctly receive the message. A broadcast to all neighbors requires a power level that can reach the farthest neighbor, which corresponds to its *Radius* (as defined in Section 2.2).

3.2 Properties of LMST

In this section, we prove some properties of LMST, including its connectivity and out-degree bound. We also determine, with the use of a probabilistic model, how often the Hello messages should be exchanged and the topology should be updated.

3.2.1 Network Connectivity

The following lemma is very important to the proof of connectivity.

Lemma 3.4. *For any edge* $(u, v) \in E(G)$, $u \Leftrightarrow v$ *in* G_{LMST}.

Proof. Let all the edges $(u, v) \in E(G)$ be sorted in ascending order of weight, i.e., $w(u_1, v_1) < w(u_2, v_2) < \cdots < w(u_l, v_l)$, where l is the total number of edges in G. We prove by induction.

1. *Basis*: The first edge (u_1, v_1) satisfies $w(u_1, v_1) = \min_{(u,v)\in E(G)}\{w(u, v)\}$. Since the shortest edge is always on the local MST, we have $u_1 \overset{LMST}{\longleftrightarrow} v_1$, which means $u_1 \Leftrightarrow v_1$.

2. *Induction*: Assume the hypothesis holds for all edges $(u_i, v_i), 1 \le i < k$, we prove $u_k \Leftrightarrow v_k$ in G_{LMST}. If $u_k \overset{LMST}{\longleftrightarrow} v_k$, then $u_k \Leftrightarrow v_k$. Otherwise, without loss of generality, assume $u_k \nrightarrow v_k$. In the local topology construction of u_k, before edge (u_k, v_k) was inspected, there must already exist a path $p = (w_0 = u_k, w_1, w_2, \ldots, w_{m-1}, w_m = v_k)$ from u_k to v_k, where $(w_i, w_{i+1}) \in E(T_{u_k}), i = 0, 1, \ldots, m-1$. Since edges are inserted in ascending order of weight, we have $w(w_i, w_{i+1}) < w(u_k, v_k)$. Applying the induction hypothesis to each pair $(w_i, w_{i+1}), i = 0, 1, \ldots, m-1$, we have $w_i \Leftrightarrow w_{i+1}$. Therefore, $u_k \Leftrightarrow v_k$.

□

Lemma 3.4 shows that, for any edge $(u, v) \in E(G)$, either $u \overset{LMST}{\longleftrightarrow} v$, or u and v are bi-directionally connected to each other in G_{LMST} via links of smaller weight.

Theorem 3.5 (Connectivity of LMST)**.** *If G is connected, then G_{LMST}, G^+_{LMST} and G^-_{LMST} are all connected.*

Proof. We only need to prove that G^-_{LMST} preserves the connectivity of G, since $E(G^-_{LMST}) \subseteq E(G_{LMST}) \subseteq E(G^+_{LMST})$. Suppose G is connected. For any two nodes $u, v \in V(G)$, there exists at least one path $p = (w_0 = u, w_1, w_2, \ldots, w_{m-1}, w_m = v)$ from u to v, where $(w_i, w_{i+1}) \in E(G), i = 0, 1, \ldots, m-1$. Since $w_i \Leftrightarrow w_{i+1}$ by Lemma 3.4, we have $u \Leftrightarrow v$ in G_{LMST}. Since p is bi-directional in G_{LMST}, the removal of uni-directional links does not affect the existence of p. Therefore, $u \Leftrightarrow v$ in G^-_{LMST}, i.e., G^-_{LMST} preserves the connectivity of G. □

3.2.2 Degree Bound

It has been observed that any minimum spanning tree of a finite set of points in the plane has a maximum logical node degree of six [80]. We prove this property independently for LMST, of which the resulting topology is generally not a tree. We only need to prove the out-degree bound for G^+_{LMST} since the out-degree bound of G_{LMST} or G^-_{LMST} can only be lower.

Lemma 3.6. *Given three nodes $u, v, p \in V(G)$ satisfying $w(u, p) < w(u, v)$ and $w(p, v) < w(u, v)$, then $u \nrightarrow v$ and $v \nrightarrow u$ in G^+_{LMST}.*

Proof. We only need to consider the case where $(u,v) \in E(G)$, since $(u,v) \notin E(G)$ would imply $u \nrightarrow v$ and $v \nrightarrow u$ in G^+_{LMST}. Consider the local topology construction of u and v. Before we insert (u,v) into T_u or T_v, the two edges (u,p) and (p,v) have already been processed since $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$. Thus $u \Leftrightarrow p$ and $p \Leftrightarrow v$ by Lemma 3.4, which means $u \Leftrightarrow v$. Therefore, (u,v) should be inserted into neither T_u nor T_v, i.e., $u \nrightarrow v$ and $v \nrightarrow u$ in G^+_{LMST}. □

The following corollary is a by-product of Lemma 3.6.

Corollary 3.7. *The topology by LMST is a subgraph of the topology by RNG, i.e.,* $G_{LMST} \subseteq G_{RNG}$*.*

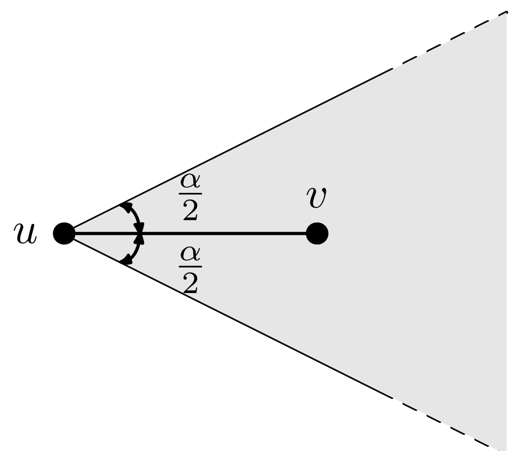


Figure 3.2: Definition of $cone(u, \alpha, v)$

Definition 3.8. *A* $Cone(u, \alpha, v)$ *is the unbounded shaded region shown in Figure 3.2.*

Theorem 3.9 (Degree Bound)**.** *The out-degree of any node in* G^+_{LMST} *is bounded by 6, i.e.,* $deg^{out}_{LMST+}(u) \leq 6, \forall u \in V(G^+_{LMST})$.

Proof. Fist we prove by contradiction that if $v \in N^{out}_{LMST+}(u)$, then there cannot exist any other node $w \in N^{out}_{LMST+}(u)$ that lies inside $Cone(u, 2\pi/3, v)$. Assume that such a node w exists, then $\angle wuv < \pi/3$. If $w(u,w) > w(u,v)$, then $\angle uvw > \pi/3 > \angle wuv$. We have $w(u,w) > w(v,w)$, which implies $u \nrightarrow w$ by Lemma 3.6. If $w(u,w) < w(u,v)$, then $\angle uwv > \pi/3 > \angle wuv$. We have $w(u,v) > w(v,w)$, which implies $u \nrightarrow v$ by Lemma 3.6. Both scenarios contradict with the assumption that $v, w \in N^{out}_{LMST+}(u)$.

Consider any node $u \in V(G^+_{LMST})$. Put the nodes in $N^{out}_{LMST+}(u)$ in order such that for the i^{th} node w_i and the j^{th} node w_j satisfying $j > i$, $w(u,w_j) > w(u,w_i)$. We have proved that $\angle w_i u w_j \geq \pi/3$, i.e.,

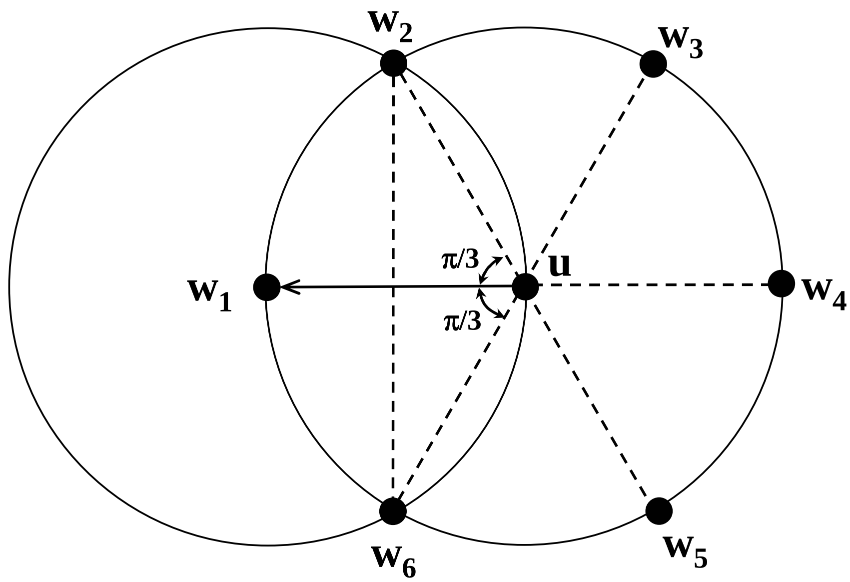


Figure 3.3: Degree bound of G^{+}_{LMST}

node w_j cannot reside inside $Cone(u, 2\pi/3, w_i)$. Therefore, node u cannot have any out-neighbor other than node w_i inside $Cone(u, 2\pi/3, w_i)$. By induction on the rank of nodes in $N^{out}_{LMST+}(u)$, the maximal number of out-neighbors that u can have is no greater than six, i.e., $deg^{out}_{LMST+}(u) \leq 6$. Figure 3.3 actually shows the only scenario where $deg^{out}_{LMST+}(u) = 6$ occurs. □

Note that what has been discussed so far is the *logical* out-degree. For an arbitrary topology, the physical out-degree cannot be bounded by a constant if all nodes use omni-directional antennas. However, with the help of directional antennas, we will be able to bound the physical out-degree given that the logical out-degree is bounded under LMST (except for some extreme cases, e.g., a large number of nodes are of the same distance from one node). When transmitting to a specific neighbor, node u adjusts its direction and limits the transmission power so that no other nodes will be affected.

As a smaller out-degree usually implies less contention and interference in wireless ad hoc networks, the out-degree bound obtained in Theorem 3.9 can be used to better design medium access algorithms. For example, several TDMA-based scheduling algorithms have been proposed to maximize the spatial reuse and minimize frame length [17] [52], most of which require that the maximum out-degree be bounded.

3.2.3 Estimation of Information Exchange Period

We now estimate the time interval between two information exchanges (i.e., two broadcasts of Hello messages) under a probabilistic model with the following assumptions:

(i) Initially all nodes are uniformly distributed within a disk of area S_0, and N, the total number of nodes in G, is known or can be estimated.

(ii) After a short time interval of length t, the location of a node u will be randomly distributed inside a disk centered at its current location, with a radius of $v_{max} \cdot t$, where v_{max} is the maximum speed of u. This is a Brownian-like mobility model that preserves the uniform node distribution [9].

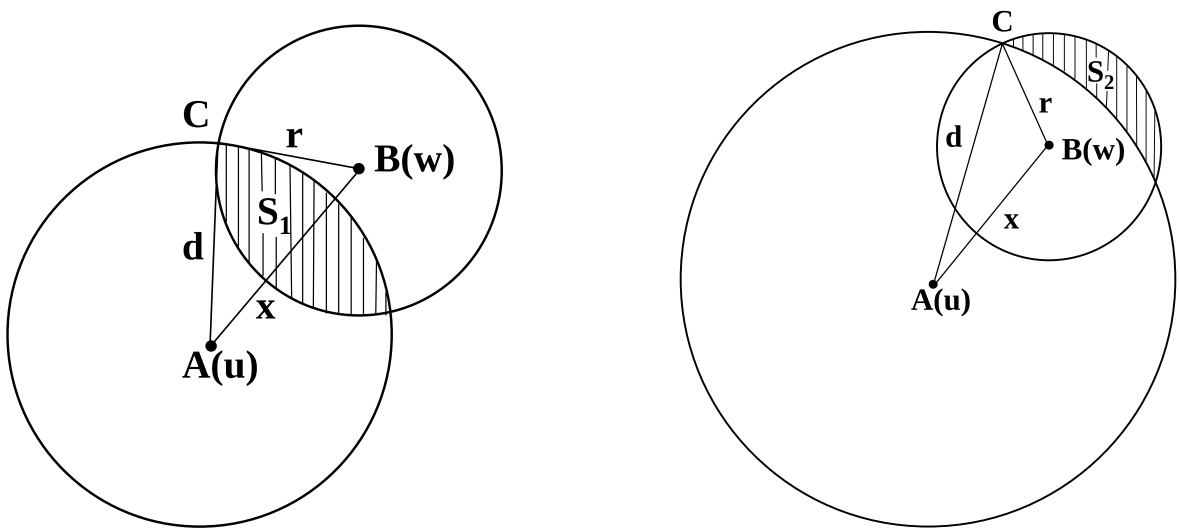
Since S_0 is relatively large and t is relatively short, the border effect can be ignored. Also, the above assumptions are made based on the notion of randomness, and may not necessarily represent the node distribution and mobility model in the real world. However, due to the fact that appropriate statistical models that characterize these distributions of interest are lacking, the above assumptions may serve to give rough estimates of information exchange periods.

Let d be the maximal transmission range for every node. Denote $D(u, d)$ as the disk of radius d centered at node u. We fix the reference frame on a node u and calculate the probabilities that a new neighbor moves into the transmission range of u and that an existing neighbor moves out of the transmission range of node u, within a time interval of t.

Probability that node w moves into the disk $D(u, d)$ As shown in Figure 3.4(a), suppose node u is located in position A, with its neighbor w in position B. The maximal transmission range of node u is $AC = d$, and the distance between nodes u and w is $x(> d)$. Let $BC = r = 2v_{max} \cdot t$. The probability that node w moves into the transmission range of node u within time t is the probability that node w moves into the disk $D(u, d)$ (i.e., the shaded area in Figure 3.4(a)) within time t. This probability can be calculated by considering the following two cases:

- *Case I*: $0 < r < 2d$. The probability, p_{join}, that node w moves into $D(u, d)$ within time t is

$$\begin{aligned} p^{I}_{join} &= \int_{d}^{d+r} \frac{2\pi x}{S_0} \frac{S_1}{\pi r^2} dx \\ &= \int_{d}^{d+r} \frac{2x S_1}{S_0 r^2} dx, \end{aligned} \tag{3.4}$$



(a) Probability that a new neighbor moves into the transmission range

(b) Probability that an existing neighbor moves out of the transmission range

Figure 3.4: Probability of topological changes

where

$$\begin{aligned} S_1 &= \alpha_1 d^2 + \alpha_2 r^2 - xr\sin\alpha_2, \\ \alpha_1 &= \angle CAB = \arccos\frac{x^2+d^2-r^2}{2xd}, \\ \alpha_2 &= \angle CBA = \arccos\frac{x^2+r^2-d^2}{2xr}. \end{aligned}$$

- *Case II*: $r \geq 2d$. The probability of interest is

$$\begin{aligned} p_{join}^{II} &= \int_d^{r-d} \frac{2\pi x}{S_0}\frac{\pi d^2}{\pi r^2}dx + \int_{r-d}^{r+d} \frac{2\pi x}{S_0}\frac{S_1}{\pi r^2}dx \\ &= \int_d^{r-d} \frac{2\pi x}{S_0}\frac{d^2}{r^2}dx + \int_{r-d}^{r+d} \frac{2xS_1}{S_0 r^2}dx \\ &= \frac{\pi d^2}{S_0 r^2}[(r-d)^2 - d^2] + \int_{r-d}^{r+d} \frac{2xS_1}{S_0 r^2}dx \\ &= \frac{\pi d^2(r-2d)}{S_0 r} + \int_{r-d}^{r+d} \frac{2xS_1}{S_0 r^2}dx. \end{aligned} \quad (3.5)$$

Thus we have

$$p_{join} = p_{join}^{I} + p_{join}^{II}. \quad (3.6)$$

Probability that node w moves out of the disk $D(u, d)$ The probability that an existing neighbor w moves out of the transmission range of node u within time t is the probability that w moves out of the disk $D(u, d)$ (i.e., into the shaded area in Figure 3.4(b)) in time t. We consider three cases:

- *Case I*: $0 < r < d$. The probability, p_{leave}, that node w moves out of $D(u, d)$ in time t is

$$\begin{aligned} p^{I}_{leave} &= \int_{d-r}^{d} \frac{2\pi x}{S_0} \frac{S_2}{\pi r^2} dx \\ &= \int_{d-r}^{d} \frac{2xS_2}{S_0 r^2} dx, \end{aligned} \tag{3.7}$$

  where

$$\begin{aligned} S_2 &= (\pi - \alpha_2) r^2 - (\alpha_1 d^2 - xr \sin\alpha_2), \\ \alpha_1 &= \angle CAB = \arccos\frac{x^2 + d^2 - r^2}{2xd}, \\ \alpha_2 &= \angle CBA = \arccos\frac{x^2 + r^2 - d^2}{2xr}. \end{aligned}$$

- *Case II*: $d \le r < 2d$. The probability of interest can be expressed as

$$\begin{aligned} p^{II}_{leave} &= \int_{0}^{r-d} \frac{2\pi x}{S_0} \frac{\pi(r^2 - d^2)}{\pi r^2} dx + \int_{r-d}^{d} \frac{2\pi x}{S_0} \frac{S_2}{\pi r^2} dx \\ &= \int_{0}^{r-d} \frac{2\pi x}{S_0} \frac{(r^2 - d^2)}{r^2} dx + \int_{r-d}^{d} \frac{2xS_2}{S_0 r^2} dx \\ &= \frac{\pi(r+d)}{S_0 r^2}(r-d)^3 + \int_{r-d}^{d} \frac{2xS_2}{S_0 r^2} dx. \end{aligned} \tag{3.8}$$

- *Case III*: $r \ge 2d$. The probability of interest can be expressed as

$$\begin{aligned} p^{III}_{leave} &= \int_{0}^{d} \frac{2\pi x}{S_0} \frac{\pi(r^2 - d^2)}{\pi r^2} dx \\ &= \frac{\pi(r^2 - d^2)d^2}{S_0 r^2}. \end{aligned} \tag{3.9}$$

Thus we have

$$p_{leave} = p^{I}_{leave} + p^{II}_{leave} + p^{III}_{leave}. \tag{3.10}$$

Estimation of information exchange periods

Given that node u has n neighbors and the total number of nodes is N, the probability that no new neighbor enters the visible neighborhood of node u is

$$p_1 = (1 - p_{join})^{N-n-1}, \tag{3.11}$$

and the probability that no neighbor leaves the visible neighborhood of node u is

$$p_2 = (1 - p_{leave})^n. \tag{3.12}$$

Thus, the probability that the visible neighborhood of node u changes is

$$p_{change} = 1 - p_1 p_2. \tag{3.13}$$

Given a predetermined probability threshold p_{th}, we can determine the topology update interval t such that $p_{change} < p_{th}$.

Note that this estimate only serves as a guideline on how to choose the interval of information exchange. To demonstrate how it is affected by the maximum speed v_{max} and the probability threshold p_{th}, we consider a scenario in which 100 nodes are randomly distributed inside a disk of radius $1000m$. The transmission range is $d_{max} = 250m$. The number of neighbors is set to 25. Figure 3.5 gives the curve of the information update period versus the maximum speed with respect to different values of p_{th}. For example, to ensure that the probability of changes in the visible neighborhood is kept below 0.2, the information update period decreases from 10.6 sec to 1.06 sec when the maximum node speed increases from $1m/s$ to $10m/s$.

3.3 Performance Evaluation

In this section, we present several sets of simulation results to evaluate the effectiveness of LMST. As R&M and CBTC come closest to our work, we compare them with LMST in the simulation study. We also use the topology derived using the maximal transmission power as the baseline. The reasons we do not compare LMST against CONNECT and COMPOW/CLUSTERPOW are two-fold: (a) CONNECT and its extension

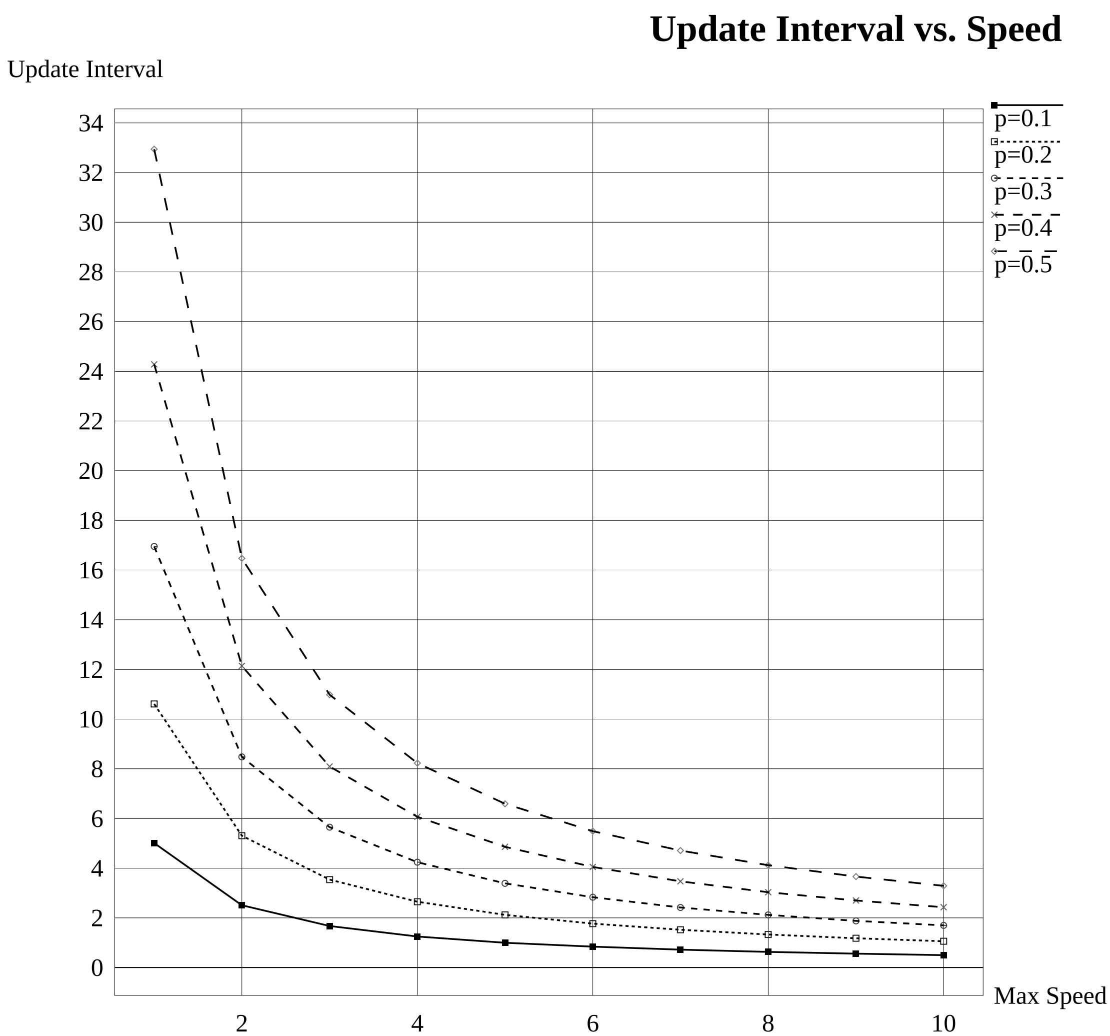


Figure 3.5: Information update period vs. maximum speed for different p_{th}

are centralized algorithms that require global information, while LMST is a localized algorithm that derives the network topology based on local information; and (b) COMPOW/CLUSTERPOW are implemented at the Network Layer and incur significant message overhead, while LMST is implemented below the Network Layer.

We will compare performance metrics of two categories: traffic-independent and traffic-dependent. The traffic-independent performance metrics used in the study are:

- Logical/physical out-degree. A smaller average out-degree usually implies less contention and interference, and better spatial reuse.

- Radius. As each node u adjusts its transmission power to the value that corresponds to the radius R_u, a smaller value of R_u implies less power consumption.

The traffic-dependent performance metrics are:

- Total data delivered (end-to-end). This serves as a good indicator of the network capacity.[1]
- Energy efficiency (bytes/Joule). Energy efficiency is defined as the total data delivered (in bytes) divided by the total energy consumption (in Joules).
- Average number of transmissions for each packet delivered. This can be interpreted as the number of times a packet has to be transmitted, on the way from its source to destination, and is loosely related to the average packet delay.

Note that the traffic-dependent performance metrics are affected by, in addition to the quality of topology control, several other factors, such as the spatial distribution of wireless devices, the MAC level contention/interference, and the routes selected by routing protocols.

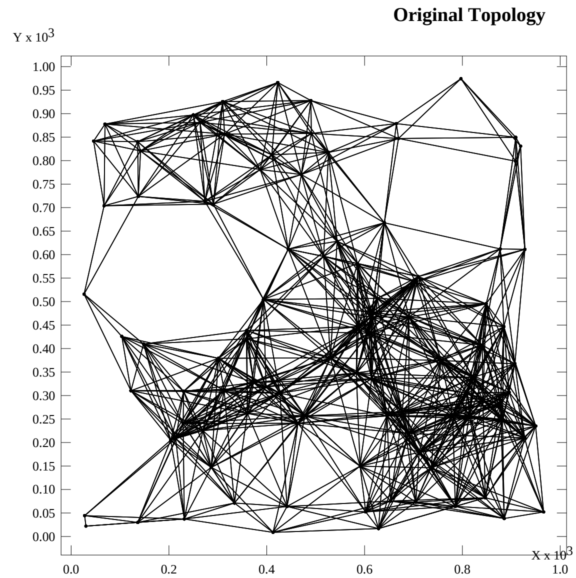
3.3.1 Traffic-independent Performance Metrics

All simulations in this section were carried out in *J-Sim* [111], a component-based, compositional network simulator written in Java (`http://www.j-sim.org`). In the simulations, nodes are randomly distributed in a $1000 \times 1000m^2$ square region. The transmission range of each node is $d_{max} = 250m$.

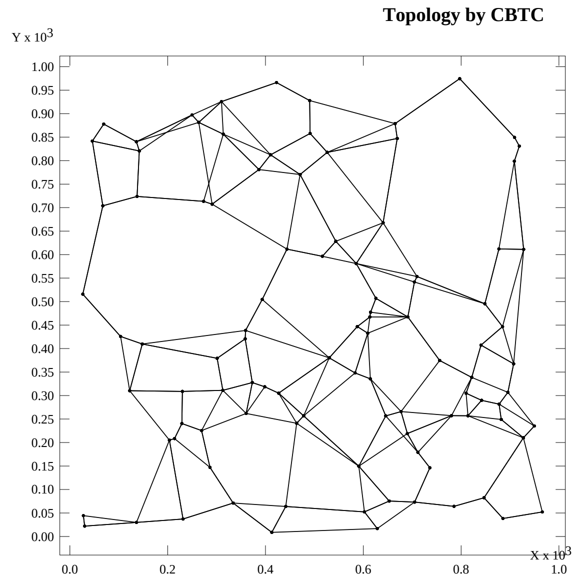
Algorithm	Maximum degree	Minimum degree	Average degree
Max trans. power	28	4	16.48
CBTC($\frac{5\pi}{6}$)	5	1	2.97
R&M(Two-Ray Ground model)	5	1	2.64
LMST	3	1	2.06
LMST with *Removal*	3	1	2.04

For a network of 100 nodes, the topology derived using the maximal transmission power, R&M (under the Two-Ray Ground Model), CBTC, and LMST with *Removal* are shown in Figure 3.6. The corresponding maximum, minimum, and average logical out-degrees are given above. R&M, CBTC and LMST all dramatically reduce the average out-degree, with LMST being clearly the best.

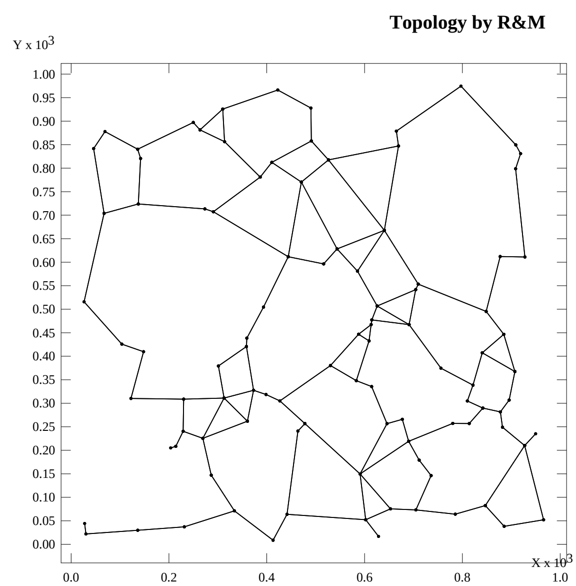
[1] Strictly speaking, the *transport capacity* of a network is defined in [37] as the sum of products of bits and the distances they travel. Here we only compare the total throughput. We will compare the transport capacity of various algorithms in Section 5.4.



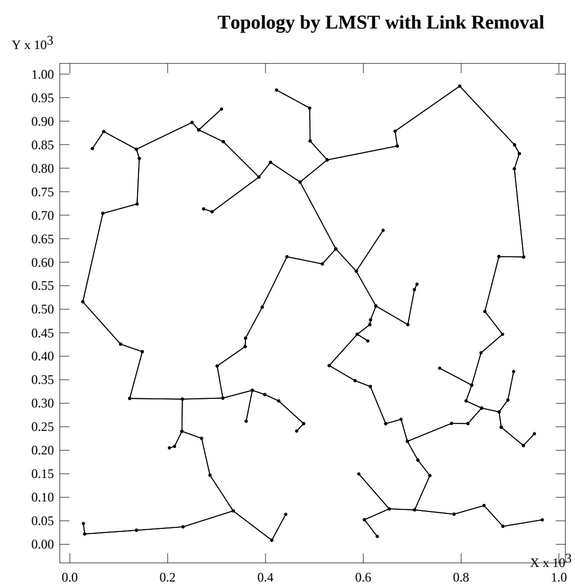
(a) The topology derived using maximal transmission power



(b) The topology generated by CBTC($\frac{5\pi}{6}$)



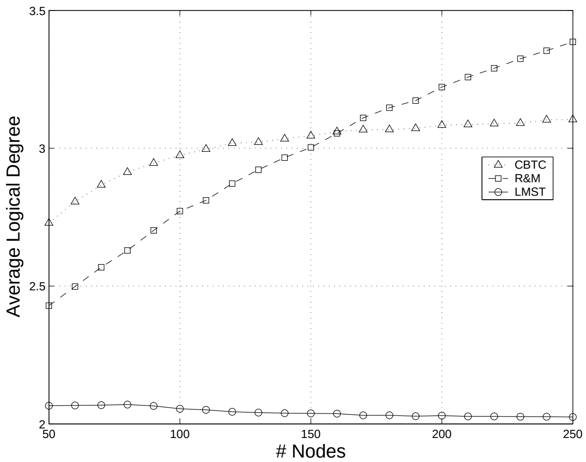
(c) The topology generated by R&M



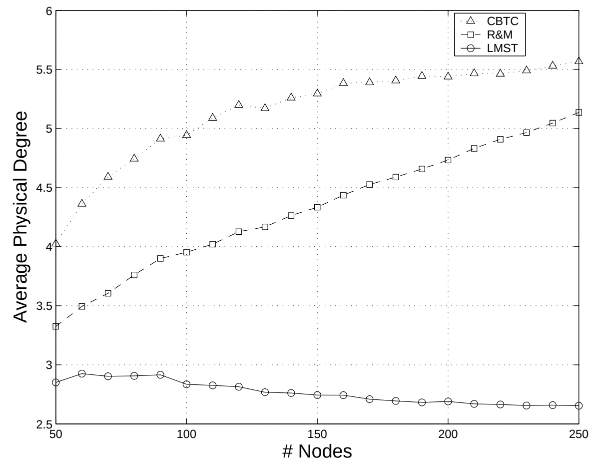
(d) The topology generated by LMST with *Removal*

Figure 3.6: Network topologies by different algorithms

In the next simulation, we vary the number of nodes in the region from 50 to 250. Each data point is the average of 100 simulation runs. The average logical and physical out-degrees for the topologies generated by R&M, CBTC($\frac{5\pi}{6}$), and LMST with *Removal* are shown in Figure 3.7 (here NONE indicates the case where no topology control is employed). Both the average logical and physical out-degrees in R&M and CBTC increase with the increase of spatial density, while that under LMST actually decreases slightly. Also, we measure the average logical out-degree for topologies generated by LMST, LMST with *Addition*



(a) Average logical out-degree



(b) Average physical out-degree

Figure 3.7: Performance comparison w.r.t. out-degree

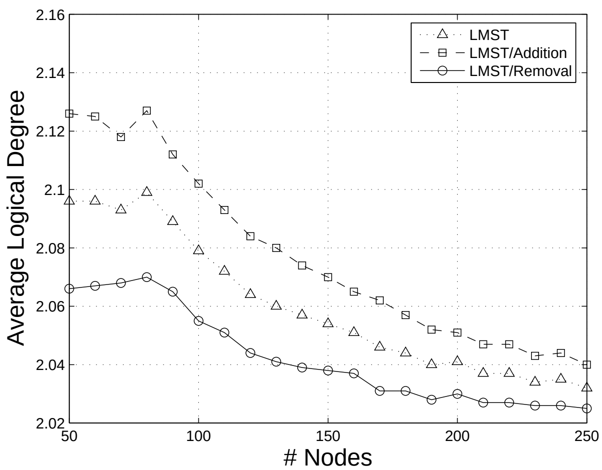


Figure 3.8: Average logical out-degree of LMST, LMST/Addition, and LMST/Removal

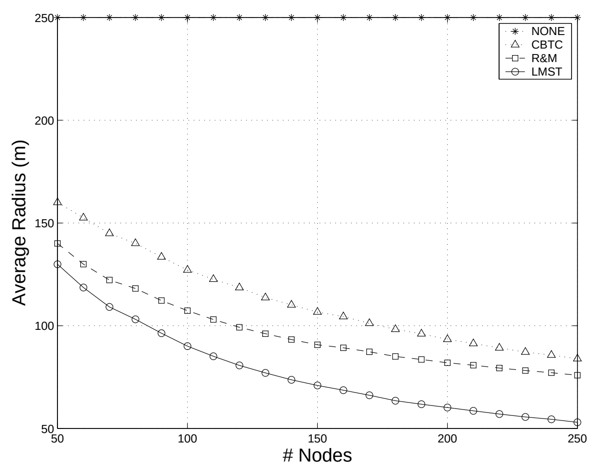


Figure 3.9: Performance comparison w.r.t. radius

and LMST with *Removal*. In Figure 3.8, it can be observed that: (1) the average logical out-degree under LMST and its two variations does not differ much, and decreases as the node density increases. This is in contrast with the fact that the average out-degree of the topology derived using the maximal transmission power increases almost linearly; (2) the average logical out-degree under LMST is very close to that of a global spanning tree, which is known to have the least average logical out-degree ($\lim_{n\to\infty} 2 - \frac{2}{n} = 2$) among all the spanning subgraphs. The average radius for the topologies under NONE, R&M, CBTC, and LMST with *Removal* is shown in Figure 3.9. The average radius under LMST is much smaller, which shows that LMST is very power-efficient.

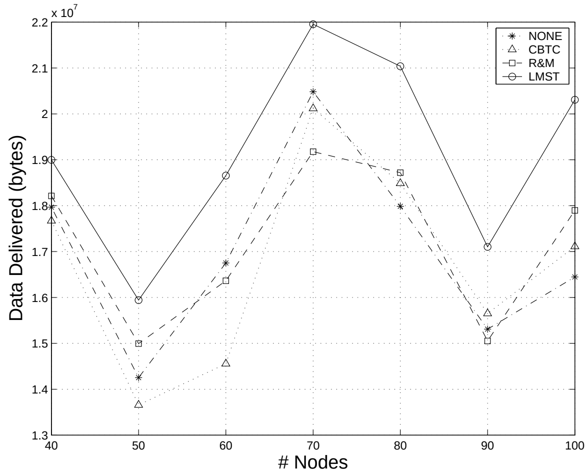
3.3.2 Traffic-dependent Performance Metrics

All simulations in this section were carried out in *ns-2* [79]. A total of n nodes are randomly distributed in a $1500 \times 200m^2$ region, with half of them being sources and the other half being destinations. On one hand, to demonstrate the effect of spatial reuse, the deployment region should be large enough as compared with the interference range (which is generally larger than the transmission range) of wireless nodes, so that there could actually be multiple concurrent successful transmission in the network. On the other hand, to ensure the connectivity of the network, a large number of nodes are required for a large deployment region, which will severely slow down the simulation. As a result, we use a rectangular deployment region, rather than a square region, so that the number of nodes needed is kept minimal and one dimension of the network is large enough.

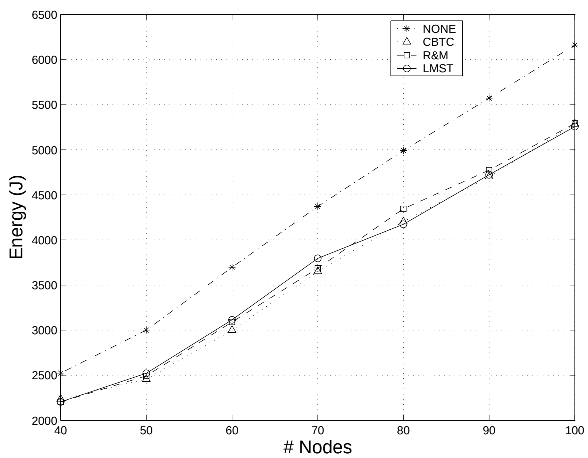
Each simulation run lasts for 200 seconds, and each data point in the figures is an average of 10 simulation runs. The number of nodes in the network, n, is varied from 40 to 150. The propagation model is the Two-Ray Ground model, the MAC protocol is IEEE 802.11 (2Mbps bandwidth), and the routing protocol is AODV. We use the energy model in *ns-2*, i.e., it takes 660mW, 395mW and 35mW for a node to transmit, receive and stay idle, respectively. Two classes of traffic sources are used: CBR and TCP with bulk FTP. The start time of each connection is chosen randomly from $[25s, 50s]$.

Performance with respect to energy efficiency We now study the impact of topology control on energy efficiency (in bytes/J), where the energy efficiency is defined to be the total end-to-end data delivery divided by the total energy consumption across the entire network. Figure 3.10 and Figure 3.11 show the total data delivered, the total energy consumption, and the energy efficiency, for TCP traffic with bulk FTP and CBR traffic, respectively. LMST delivers the most amount of data, while the other three do not differ much in the amount of data delivered. Moreover, LMST is more energy-efficient.

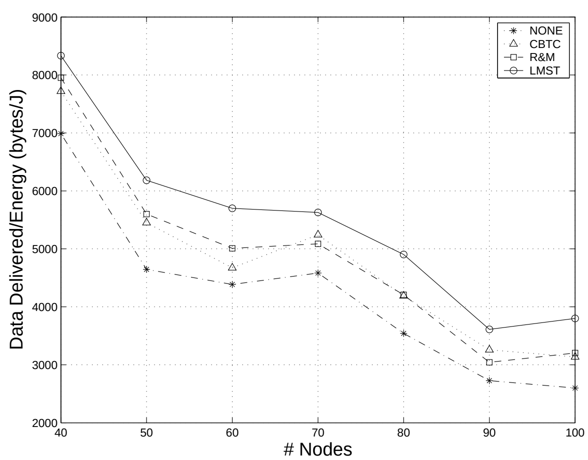
Performance with respect to # transmissions each packet incurs Figure 3.12 shows the average number of transmissions for each packet delivered. This can be interpreted as the number of times each packet has to be transmitted, hop by hop, on its way from the source to the destination. As a topology control algorithm constrains a node from transmitting using the maximal transmission power, it is usually believed that packets traverse more hops (and hence incurs more number of transmissions) on the topology generated by a topology control algorithm. Although this conclusion may be true, topology control does not neces-



(a) Total throughput (bytes)

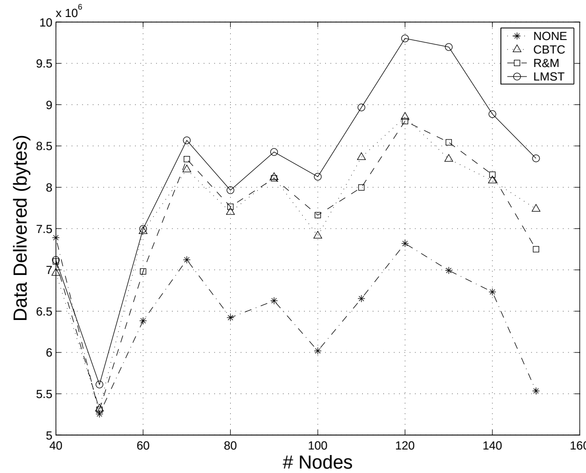


(b) Total energy consumption (Joule)

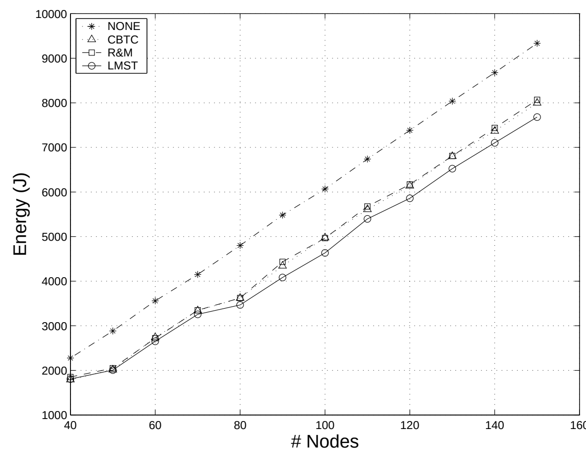


(c) Energy efficiency (bytes/J)

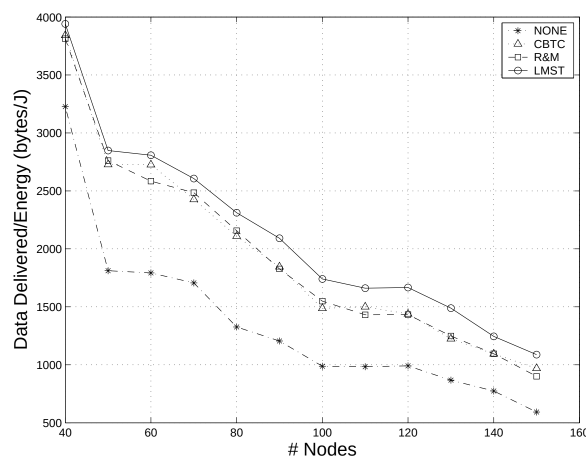
Figure 3.10: Network capacity and energy efficiency under TCP traffic with bulk FTP



(a) Total throughput (bytes)

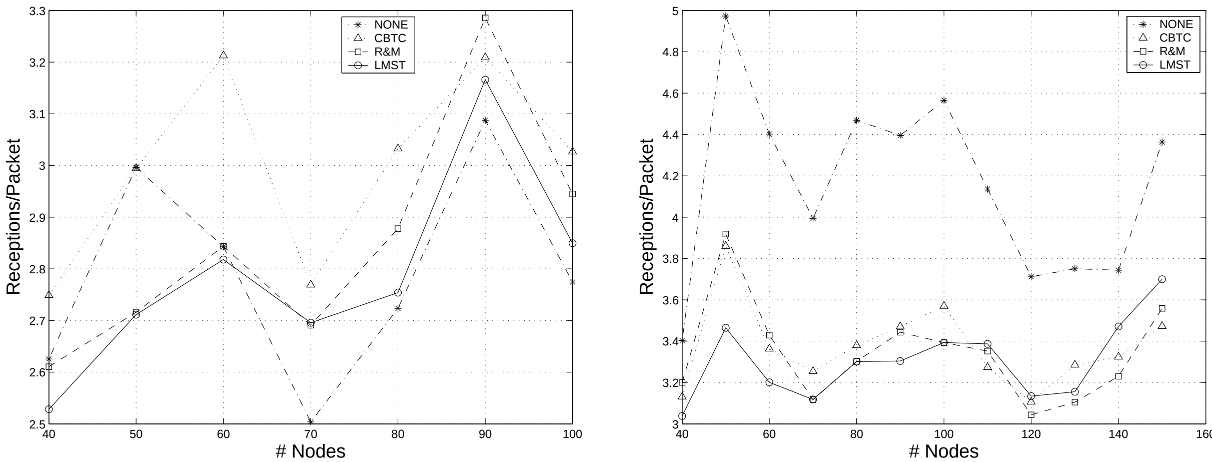


(b) Total energy consumption (Joule)



(c) Energy efficiency (bytes/J)

Figure 3.11: Network capacity and energy efficiency under CBR traffic



(a) # Transmissions/packet under TCP traffic (b) # Transmissions/packet under CBR traffic

Figure 3.12: Number of transmissions per packet delivered

sarily introduce more transmissions. As shown in Figure 3.12, in the case of TCP traffic, LMST incurs the least number of transmission among all three topology control algorithms and performs slightly worse than NONE. In the case of CBR traffic, all three topology control algorithms outperform NONE. We believe this is because with topology control, the medium is shared in a more efficient manner so that data packets do not encounter excessive medium contention/collision and can be delivered more quickly.

To summarize, LMST outperforms the other topology control schemes in terms of the total amount of data delivered and energy efficiency. In the case of high network loads, LMST can achieve better spatial reuse and delivers packets more quickly.

3.4 Conclusions

We presented LMST, a localized MST-based topology control algorithm for wireless ad hoc networks with limited mobility. As each node builds its local minimum spanning tree independently using locally collected information, the algorithm incurs less message overhead/delay in deriving the topology. Local repair can be easily made in the case of mobility. We also prove that the algorithm exhibits several desirable properties: (1) the topology derived preserves the network connectivity; (2) the out-degree of any node in the topology is bounded by 6; and (3) the topology can be transformed into one with bi-directional links (without impairing the network connectivity) after removal of all uni-directional links.

In the simulation study, we show that the topology generated by LMST achieves a small average out-degree (which is very close to the theoretical bound), and a small average radius. The former reduces the MAC-level contention, while the latter implies that only small transmission power is needed to maintain connectivity. Simulation results also indicate that LMST outperforms the other known topology control algorithms in the total amount of data delivered (in bytes), the energy efficiency (in bytes/J), and the end-to-end delay. In particular, the simulation results invalidate the common belief that as packets traverse more hops on the topology generated by a topology control algorithm, the number of transmissions for each packet delivered is also larger. With topology control, the medium is shared in a more efficient manner, data packets do not encounter excessive medium contention/collision and hence the number of transmissions per packet is not necessarily larger.

Chapter 4

Localized Topology Control in Heterogeneous Networks

The assumption of homogeneous wireless networks does not always hold in reality due to various reasons. First, even wireless devices of the same type may have slightly different transmission ranges because of manufacturing variations. Second, there exist networks with devices of dramatically different capabilities, e.g., the communication network in the Future Combat System [101].

In this chapter, we will instead consider a heterogeneous wireless network where the transmission range of each node may be non-uniform. Hence the network topology G is a directed graph as defined in Section 2.2. Let r_u be the transmission range of node u, and r_{min} and r_{max} be the smallest and the largest transmission ranges in the network, respectively. As will be shown in the next section, most existing topology control algorithms may render disconnectivity when applied directly to heterogeneous networks.

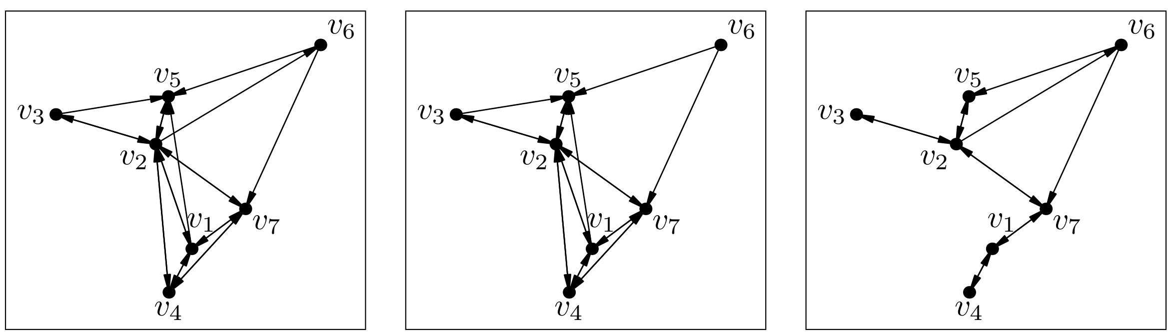
We propose two localized topology control algorithms for heterogeneous wireless ad hoc networks: *Directed Relative Neighborhood Graph* (DRNG) and *Directed Local Spanning Subgraph* (DLSS) [69]. We prove that (1) the topology generated by DRNG or DLSS preserves network connectivity; (2) the out-degree of any node in the resulting topology by DRNG or DLSS is bounded by a constant; and (3) the topology generated by DRNG or DLSS preserves network bi-directionality.

Simulation results indicate that compared with existing topology control algorithms that can be applied to heterogeneous networks, DRNG and DLSS generate network topologies with smaller average node degrees (both logical and physical), smaller average link lengths, and smaller average radii. To the best of our knowledge, this is among the first efforts to address the connectivity and bi-directionality issues in heterogeneous wireless networks.

The rest of this chapter is organized as follows. We first give examples in Section 4.1 to show why existing algorithms cannot be directly applied to heterogeneous networks. Then we present both DRNG and DLSS algorithms in Section 4.2, and prove several of their useful properties in Section 4.3. Finally, we evaluate the performance of the proposed algorithms in Section 4.4 and conclude this chapter in Section 4.5.

4.1 Motivations

Most existing topology control algorithms (except [96, 114]) assume homogeneous wireless nodes with uniform transmission ranges. When directly applied to heterogeneous networks, these algorithms may render disconnectivity. In this section, we give several examples to motivate the need for new topology control algorithms for heterogeneous networks.



(a) Original topology (without topology control) is strongly connected

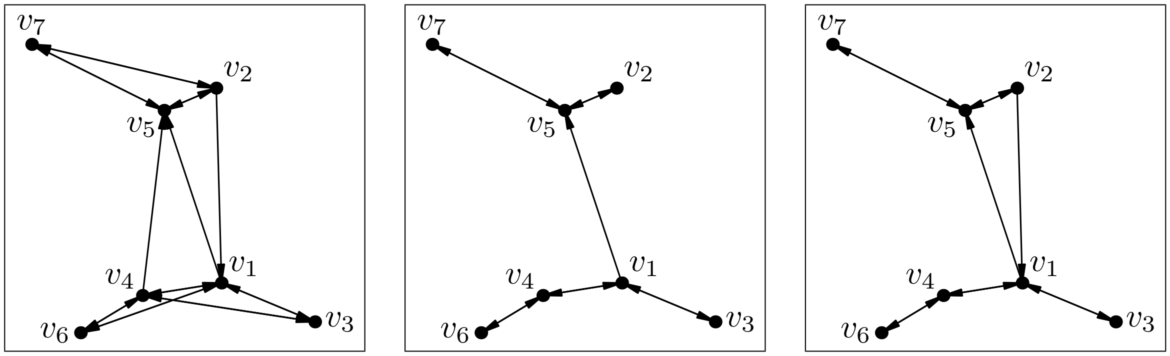
(b) Topology by CBTC($\frac{2\pi}{3}$) without optimization is not strongly connected: there is no path from v_2 to v_6

(c) Topology by DLSS is strongly connected

Figure 4.1: CBTC($\frac{2\pi}{3}$) may render disconnectivity in heterogeneous networks

As shown in Figure 4.1 (where the arrows in the figures indicate the direction of the links), the network topology generated by CBTC($\frac{2}{3}\pi$) [63] (without optimization) may not preserve the connectivity in a heterogeneous network. There is no path from v_2 to v_6 due to the loss of edge (v_2, v_6), which is discarded by v_2 since v_5 and v_7 have already provided the necessary coverage. CBTC($\frac{5}{6}\pi$) also has the same problem.

Similarly we show in Figure 4.2 that the network topology generated by RNG may be disconnected for heterogeneous network. There is no path from v_5 to v_1 due to the loss of edge (v_2, v_1), which is discarded since $|(v_5, v_1)| < |(v_2, v_1)|$, and $|(v_5, v_2)| < |(v_2, v_1)|$. Since RNG is defined for undirected graphs only, we may tailor its definition for directed graphs.

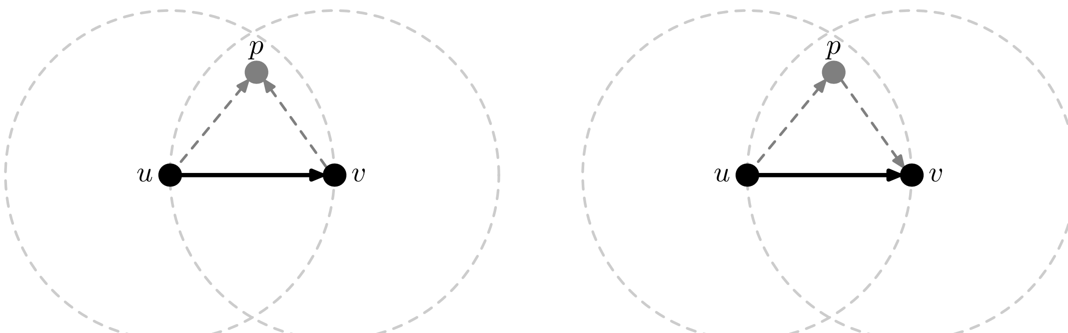


(a) Original topology (without topology control) is strongly connected

(b) Topology by RNG is not strongly connected: there is no path from v_5 to v_1

(c) Topology by DLSS is strongly connected

Figure 4.2: RNG may render disconnectivity in heterogeneous networks



(a) *Modified Relative Neighborhood Graph* (MRNG)

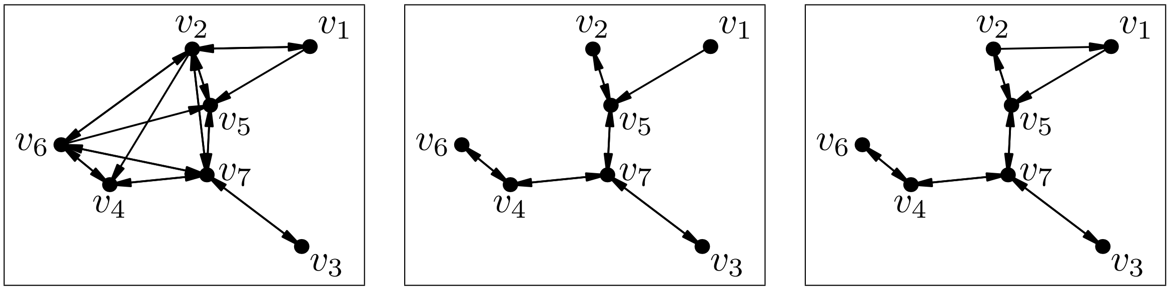
(b) *Directed Relative Neighborhood Graph* (DRNG)

Figure 4.3: The definition of MRNG and DRNG (to be defined in Section 4.2)

Definition 4.1 (MRNG)**.** *For* Modified Relative Neighborhood Graph *(MRNG),* $u \xrightarrow{MRNG} v$ *if and only if there does not exist a third node* p *such that* $w(u,p) < w(u,v), d(u,p) \leq r_u$ *and* $w(p,v) < w(u,v), d(v,p) \leq r_v$ *(Figure 4.3(a)).*

As shown in Figure 4.4, the topology generated by MRNG may still be disconnected. There is no path from v_5 to v_1 due to the loss of edge (v_2, v_1), which is discarded since $|(v_2, v_5)| < |(v_2, v_1)|$, and $|(v_1, v_5)| < |(v_2, v_1)|$.

One possible extension of LMST is for each node to build a local *directed* minimum spanning tree [10, 15, 18, 29, 108] and keep only immediate neighbors on the tree. Unfortunately, as shown in Figure 4.5, the resulting topology may not preserve the strong connectivity.

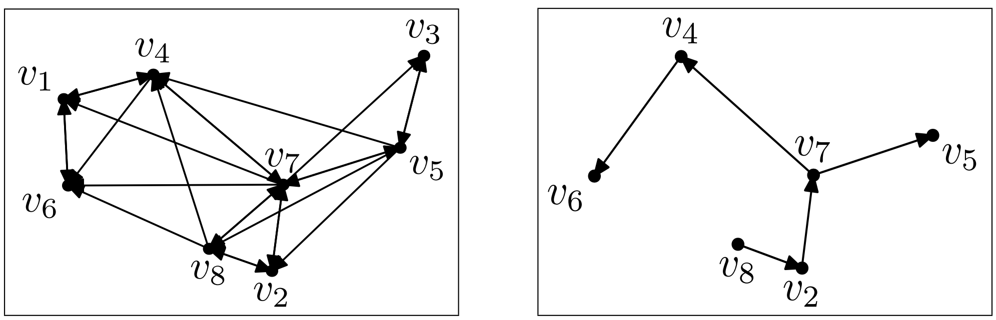


(a) Original topology (without topology control) is strongly connected

(b) Topology by MRNG is not strongly connected: there is no path from v_5 to v_1

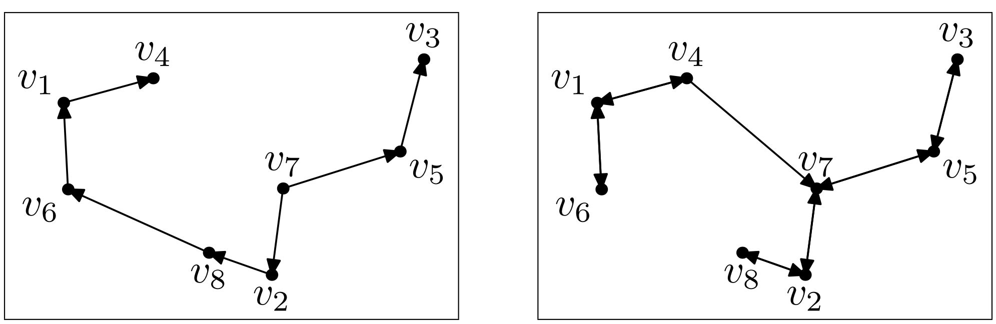
(c) Topology by DLSS is strongly connected

Figure 4.4: MRNG may render disconnectivity in heterogeneous networks



(a) Original topology (without topology control) is strongly connected

(b) The local directed MST rooted at v_8



(c) The local directed MST rooted at v_7

(d) The resulting topology is not strongly connected: there is no path from v_7 to v_4

Figure 4.5: Simple extension of LMST may render disconnectivity in heterogeneous networks

4.2 Localized Algorithms: DRNG and DLSS

In this section, we propose two localized topology control algorithms for heterogeneous wireless ad hoc networks with non-uniform transmission ranges: DRNG (Directed Relative Neighborhood Graph) and DLSS (Directed Local Spanning Subgraph). Neither DRNG nor DLSS relies on any specific radio propagation model. Being fully localized, they adapt well to mobility and incur less overhead.

In both algorithms, the network topology is constructed by having each node adjust its transmission power based on locally collected information. Both algorithms are composed of three phases:

1. *Information Collection*: each node collects the information of its visible neighborhood;

2. *Topology Construction*: based on the information of the visible neighborhood, each node defines the proper set of neighbors for the final topology.

3. *Construction of Topology with Only Bi-Directional Links* (Optional): each node adjusts its set of neighbors to make sure that all the edges are bi-directional.

4.2.1 Information Collection

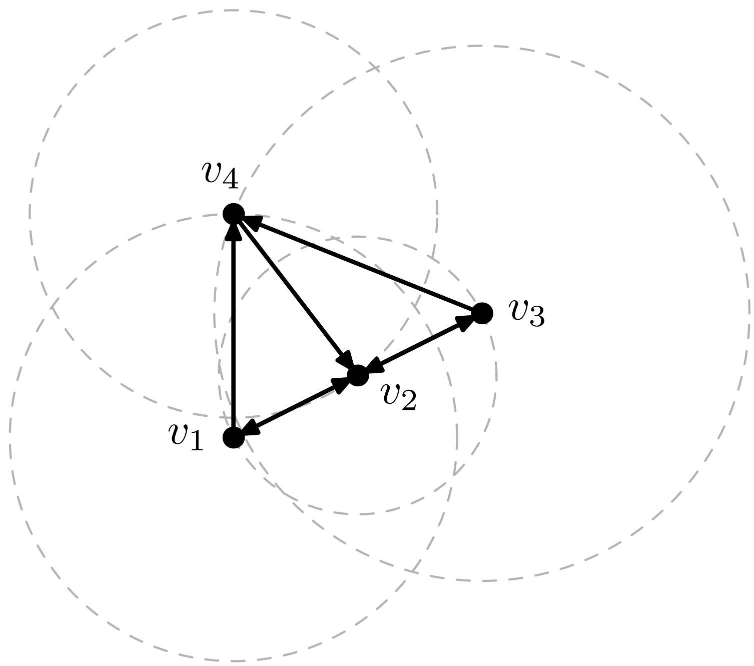


Figure 4.6: A heterogeneous network

In the stage of information collection, every node u collects the information of its visible neighborhood N_u^V. For homogeneous networks, the information can be obtained locally if each node broadcasts periodically a Hello message using its maximal transmission power, as discussed in Section 3.1.1. For heterogeneous networks, it may be insufficient for each node to broadcast periodically Hello messages. For example, as shown in Figure 4.6, v_1 is unable to know the position of v_4 since v_4 cannot reach v_1 directly.

For the ease of presentation, we assume for now that every node u obtains its N_u^V by the end of the first phase. We will revisit this issue in Section 4.3.4.

4.2.2 Topology Construction

After each node has obtained its visible neighborhood, the neighbor relation in both algorithms can be defined as follows.

Definition 4.2 (Neighbor Relation in DRNG)**.** *For* Directed Relative Neighborhood Graph *(DRNG), $u \xrightarrow{DRNG} v$ if and only if $v \in N_u^V$ and there does not exist a third node $p \in N_u^V$ such that $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v), d(p,v) \leq r_p$ (see Figure 4.3(b)).*

Procedure: DLSS(u)
Input: G_u^V, the induced subgraph of G that spans the visible neighborhood of u;
Output: $S_u = (V(S_u), E(S_u))$, the local spanning subgraph of G_u^V;
begin
    $V(S_u) := V$, $E(S_u) := \emptyset$;
    Sort all edges in $E(G_u^V)$ in the ascending order of weight
    for each edge (u_0, v_0) in the order
        if u_0 is not connected to v_0 in S_u
            $E(S_u) := E(S_u) \cup \{(u_0, v_0)\}$;
        endif
    end
end

Figure 4.7: DLSS algorithm

Definition 4.3 (Neighbor Relation in DLSS)**.** *For* Directed Local Spanning Subgraph *(DLSS), $u \xrightarrow{DLSS} v$ if and only if $(u,v) \in E(S_u)$, where S_u is the output of the DLSS algorithm shown in Figure 4.7. Hence node v is a out-neighbor of node u if and only if node v is an immediate neighbor on u's directed local spanning graph S_u.*

DRNG and DLSS are natural extensions of RNG and LMST to heterogeneous networks, respectively. For DLSS, instead of computing a directed local MST that minimizes the *total* cost of all the edges in the subgraph and is shown to be unable to preserve connectivity Section 4.1, each node computes a directed local subgraph (Figure 4.7) that minimizes the *maximum* cost among all edges in the subgraph.

4.2.3 Construction of Topology with Only Bi-directional Links

Since the final topology is determined independently by each node, some links in G_{DLSS} or G_{DRNG} may be uni-directional, as illustrated in Figure 4.11. We can apply either *Addition* or *Removal* to G_{DLSS} and G_{DRNG} to obtain bi-directional topologies. More discussions will be given in Section 4.3.2.

4.3 Properties of DRNG and DLSS

In this section, we prove the connectivity and bi-directionality of DRNG and DLSS, and derive the bound on the node degree. Then we discuss how to deal with the problem that arises in obtaining the visible neighborhood in heterogeneous networks,

4.3.1 Connectivity

Lemma 4.4. *For any edge $(u, v) \in E(G)$, we have $u \Rightarrow v$ in G_{DLSS}.*

Proof. Let all the edges $(u, v) \in E(G)$ be sorted in ascending order of weight, i.e., $w(u_1, v_1) < w(u_2, v_2) < \ldots < w(u_l, v_l)$, where l is the total number of edges in G. We prove by induction.

- *Basis*: The first edge (u_1, v_1) satisfies $w(u_1, v_1) = \min_{(u,v)\in E(G)}\{w(u, v)\}$. According to the algorithm in Figure 4.7, (u_1, v_1) and (v_1, u_1) will be included in G_{DLSS}, i.e., $u_1 \xleftrightarrow{DLSS} v_1$.

- *Induction*: Assume the hypothesis holds for all edges $(u_i, v_i), 1 \leq i < k$, we prove $u_k \Rightarrow v_k$ in G_{DLSS}. If $u_k \xrightarrow{DLSS} v_k$, then $u_k \Rightarrow v_k$. Otherwise in the local topology construction of v, before edge (u_k, v_k) was inserted into S_{u_k}, there must already exist a path $p = (w_0 = u_k, w_1, w_2, \cdots, w_{m-1}, w_m = v_k)$ from u_k to v_k, where $(w_i, w_{i+1}) \in E(S_{u_k}), i = 0, 1, \cdots, m-1$. Since edges are inserted in ascending order of weight, we have $w(w_i, w_{i+1}) < w(u_k, v_k)$. Applying the induction hypothesis to each pair $(w_i, w_{i+1}), i = 0, 1, \cdots, m-1$, we have $w_i \Rightarrow w_{i+1}$, and consequently $u_k \Rightarrow v_k$.

□

Note that we can only prove that $u \Rightarrow v$ in Lemma 4.4, since $u_k \rightarrow v_k$ does not guarantee that $v_k \rightarrow u_k$.

Theorem 4.5 (Connectivity of DLSS). *G_{DLSS} preserves the connectivity of G, i.e., G_{DLSS} is strongly connected if G is strongly connected.*

Proof. Suppose G is strongly connected. For any two nodes $u, v \in V(G)$, there exists at least one path $p = (w_0 = u, w_1, w_2, \cdots, w_{m-1}, w_m = v)$ from u to v, where $(w_i, w_{i+1}) \in E(G), i = 0, 1, \cdots, m-1$. Since $w_i \Rightarrow w_{i+1}$ by Lemma 4.4, we have $u \Rightarrow v$. □

Lemma 4.6. *Given three nodes* $u, v, p \in V(G_{DLSS})$ *satisfying* $w(u,p) < w(u,v)$ *and* $w(p,v) < w(u,v)$, $d(p,v) \leq r_p$, *then* $u \nRightarrow v$ *in* G_{DLSS}.

Proof. We only need to consider the case where $d(u,v) \leq r_u$, since $d(u,v) > r_u$ would imply $u \nRightarrow v$. Consider the local topology construction of u. Before we insert (u,v) into S_u, the two edges (u,p) and (p,v) have already been processed since $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$. Thus $u \Rightarrow p$ and $p \Rightarrow v$, which means $u \Rightarrow v$. Therefore, (u,v) should not be inserted into S_u according to the algorithm in Figure 4.7, i.e., $u \nRightarrow v$ in G_{DLSS}. □

Lemma 4.7. *The edge set of* G_{DLSS} *is a subset of the edge set of* G_{DRNG}, *i.e.,* $E(G_{DLSS}) \subseteq E(G_{DRNG})$.

Proof. For any edge $(u,v) \notin E(G_{DRNG})$, there must exist a node p such that $w(u,p) < w(u,v), d(u,p) \leq r_u$ and $w(p,v) < w(u,v), d(p,v) \leq r_p$. By Lemma 4.6, $u \nRightarrow v$ in G_{DLSS}, i.e., $(u,v) \notin E(G_{DLSS})$. Therefore, $E(G_{DLSS}) \subseteq E(G_{DRNG})$. □

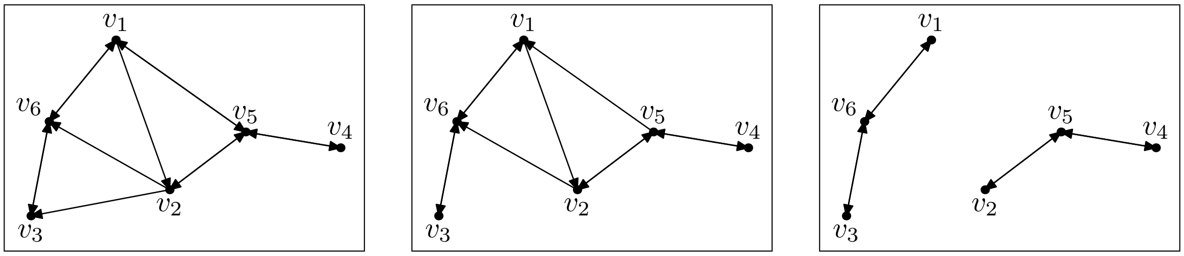
The following theorem is a direct result of Theorem 4.5 and Lemma 4.7.

Theorem 4.8 (Connectivity of DRNG). *If* G *is strongly connected, then* G_{DRNG} *is also strongly connected.*

4.3.2 Bi-directionality

Now we discuss the bi-directionality property of DRNG and DLSS. Since *Addition* may not always result in bi-directional topologies, we can apply *Removal* to topologies generated by DRNG and DLSS. It turns out that the simple *Removal* operation may lead to disconnectivity. Examples are given in Figure 4.8 and Figure 4.9 to show that DRNG and DLSS with *Removal* may result in disconnectivity.

In general, G may not be bi-directional if the maximal transmission ranges are non-uniform. Since the maximal transmission range can not be increased, it may be impossible to find a bi-directional connected subgraph of G in some cases. An example is given in Figure 4.6: v_1 can reach v_2 and v_4, v_2 can reach v_1 and v_3, v_3 can reach v_2 and v_4, and v_4 can reach v_2 only. *Addition* does not lead to bi-directionality since all edges entering or leaving v_4 are uni-directional with all nodes already transmitting with their maximal

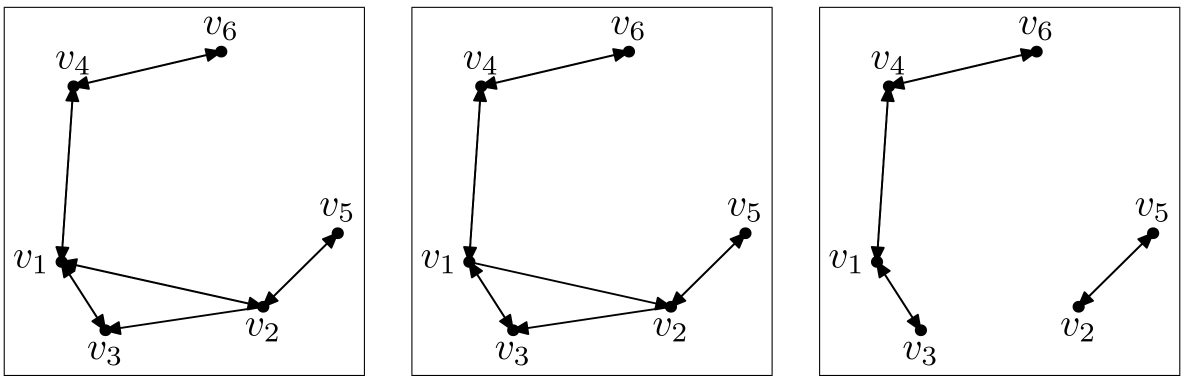


(a) Original topology (without topology control) is strongly connected

(b) Topology by DLSS is strongly connected

(c) Topology by DLSS with *Removal* is not strongly connected: there are 2 components

Figure 4.8: DLSS with *Removal* may result in disconnectivity



(a) Original topology (without topology control) is strongly connected

(b) Topology by DRNG is strongly connected

(c) Topology by DRNG with *Removal* is not strongly connected: there are 2 components

Figure 4.9: DRNG with *Removal* may result in disconnectivity

power. *Removal* also does not work since it will partition the network. In this example, although the graph G is strongly connected, its spanning subgraph cannot be both connected and bi-directional.

Now we show that bi-directionality can be ensured if the original topology is both strongly connected and bi-directional.

Theorem 4.9. *If the original topology G is strongly connected and bi-directional, then G_{DLSS} and G_{DRNG} are also strongly connected and bi-directional after* Addition *or* Removal.

Proof. We have $E(G^-_{DLSS}) \subseteq E(G^+_{DLSS})$ and $E(G^-_{DRNG}) \subseteq E(G^+_{DRNG})$. We also have $E(G^-_{DLSS}) \subseteq E(G^-_{DRNG})$ since $E(G_{DLSS}) \subseteq E(G_{DRNG})$. Therefore, we only need to prove the case for G^-_{DLSS}.

In the *Induction* step in Lemma 4.4, the only reason we cannot prove that $u_k \overset{DLSS}{\longleftrightarrow} v_k$ is that edge (v_k, u_k) may not exist. Given that G is bi-directional, we are able to prove that $u_k \overset{DLSS}{\longleftrightarrow} v_k$. Hence for any edge $(u, v) \in E(G)$, we have $u \Leftrightarrow v$ in G_{DLSS}. The removal of asymmetric edges in G_{DLSS} does not affect this property. Therefore, G^{-}_{DLSS} is bi-directionally connected. $\square$

4.3.3 Degree Bound

In this section, we derive the bound on the node degree in the topology by DRNG and DLSS. It has been observed that any minimum spanning tree of a simple undirected graph in the plane has a maximum out-degree of 6 [80]. However, this bound does not hold for directed graphs. An example is shown in Figure 4.10, where node u has 18 out-neighbors. In Figure 4.10, the transmission range of u is r_{max} and the transmission range for all other nodes is r_{min}, where $r_{max} = 2(r_{min} + \epsilon)$, $\epsilon > 0$. All nodes are so arranged that the distance between any node and its closest neighbor is $r_{min} + \epsilon$. Therefore, the only links are those from u to the other nodes. Since it is impossible to relay packets, u has to use its maximal transmission power and keeps all 18 neighbors.

The following lemma is a direct result of the definition of DRNG.

Lemma 4.10. *Given three nodes $u, v, p \in V(G_{DRNG})$ satisfying $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$, $d(p,v) \leq r_p$, then $u \nrightarrow v$ in G_{DRNG}.*

The following corollary is a direct result of Lemma 4.6 and Lemma 4.10.

Corollary 4.11. *If v is an out-neighbor of u in G_{DLSS} or G_{DRNG}, and $d(u,v) \geq r_{min}$, then u can not have any other out-neighbor inside $Disk(v, r_{min})$.*

Base on Lemma 4.6 and Lemma 4.10, the following theorem can be proved by following the arguments in Theorem 3.9.

Theorem 4.12. *For any node u in G_{DLSS} or G_{DRNG}, the number of out-neighbors inside $Disk(u, r_{min})$ is at most 6.*

Proof. Let $N^{out}(u)$ be the set of out-neighbors of u in G_{DLSS} or G_{DRNG} that are inside $Disk(u, r_{min})$. Let the nodes in $N^{out}(u)$ be ordered such that for the i^{th} node w_i and the j^{th} node w_j satisfying $j > i$, $w(u, w_j) > w(u, w_i)$. By Lemma 4.6 and Lemma 4.10, we have $w(u, w_j) \leq w(w_i, w_j)$ (otherwise $u \nrightarrow w_j$). Thus $\angle w_i u w_j \geq \pi/3$, i.e., node w_j cannot reside inside $Cone(u, 2\pi/3, w_i)$. Therefore, node u cannot have out-neighbors other than node w_i inside $Cone(u, 2\pi/3, w_i)$. By induction on the rank of nodes in $N^{out}(u)$, the maximal number of out-neighbors that u can have is at most 6. $\square$

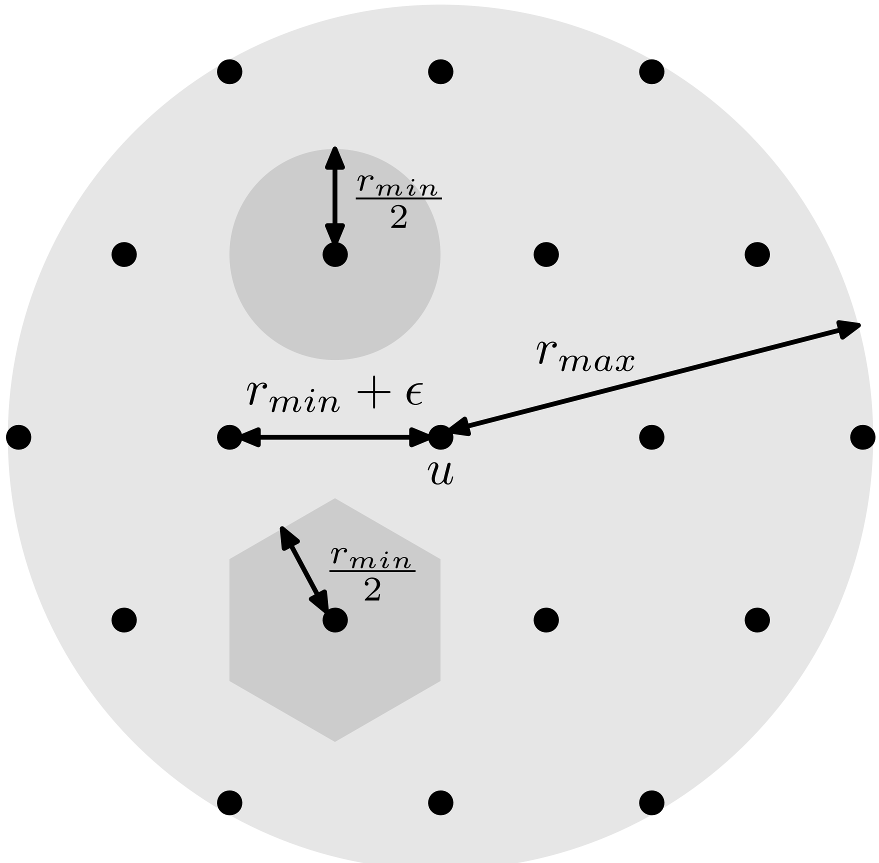


Figure 4.10: Out-degree in a heterogeneous network can be very large

Theorem 4.13 (Out-Degree Bound)**.** *The out-degree of any node in G_{DLSS} or G_{DRNG} is bounded by a constant that depends only on r_{max} and r_{min}.*

Proof. By Theorem 4.12, for any node u in G_{DLSS} or G_{DRNG}, there are at most 6 out-neighbors inside $Disk(u, r_{min})$. And by Corollary 4.11, the set of disks $\{Disk(v, \frac{r_{min}}{2}) : v \in N^{out}(u), v \notin Disk(u, r_{min})\}$ are disjoint. Therefore, the total number of out-neighbors of u is bounded by:

$$c_1 = 6 + \left\lceil \frac{\pi[(r_{max} + \frac{r_{min}}{2})^2 - (\frac{r_{min}}{2})^2]}{\pi(\frac{r_{min}}{2})^2} \right\rceil = 4\lceil \beta(\beta + 1) \rceil + 6, \tag{4.1}$$

where $\beta = \frac{r_{max}}{r_{min}}$. In Figure 4.10, the maximum out-degree of u is achieved if $\epsilon \to 0$. We can further tighten the bound. Since the hexagonal area (as shown in Figure 4.10) centered at every out-neighbor of u is disjoint with each other, the total number of out-neighbors of u is bounded by:

$$c_2 = \left\lceil \frac{\pi(r_{max} + \frac{r_{min}}{\sqrt{3}})^2}{\frac{\sqrt{3}}{2} r_{min}^2} \right\rceil - 1 = \left\lceil \frac{2\pi}{\sqrt{3}}(\beta + \frac{1}{\sqrt{3}})^2 \right\rceil - 1. \tag{4.2}$$

□

Theorem 4.14 (In-Degree Bound)**.** *The in-degree of any node in G_{DLSS} or G_{DRNG} is bounded by 6.*

Proof. Let $N^{in}(u)$ be the set of in-neighbors of u in G_{DLSS} or G_{DRNG}. Sort the nodes in $N^{in}(u)$ such that for the i^{th} node w_i and the j^{th} node w_j satisfying $j > i$, $w(w_j, u) > w(w_i, u)$. By Lemma 4.6 and Lemma 4.10, we have $w(w_j, u) \leq w(w_i, w_j)$ (otherwise $w_j \nrightarrow u$). Thus $\angle w_i u w_j \geq \pi/3$, i.e., node w_j cannot reside inside $Cone(u, 2\pi/3, w_i)$. Therefore, node u cannot have in-neighbors other than node w_i inside $Cone(u, 2\pi/3, w_i)$. By induction on the rank of nodes in $N^{in}(u)$, the maximal number of in-neighbors that u can have is at most 6. □

The bound given in Theorem 4.12 is the same for DRNG and DLSS, but the out-degree of any node in G_{DLSS} is always smaller than that in G_{DRNG} since $E(G_{DLSS}) \subseteq E(G_{DRNG})$. This bound is quite large since it describes the worst scenario. The average out-degree of nodes is actually not as large. In particular, since $\sum_{v \in V} deg^{in}(v) = \sum_{v \in V} deg^{out}(v)$, we have

$$E[deg^{out}(v)] = \frac{1}{n} \sum_{v \in V} deg^{in}(v) \leq \frac{1}{n} \cdot 6n = 6. \tag{4.3}$$

We will also show in Section 4.4 that the average maximum out-degree is far less than the bound for networks with randomly distributed nodes.

4.3.4 Obtaining Visible Neighborhood in Heterogeneous Networks

Each node at least has to know the information of its visible neighborhood to be able to preserve the network connectivity. As mentioned in Section 4.2, for a heterogeneous network, it is sometimes insufficient for each node u to broadcast its own position information to all the other nodes within r_u (Figure 4.6). This problem is common to any distributed/localized topology control algorithm.

For a general heterogeneous network, there is no guarantee that this problem can be solved locally. The only solution is for each node u to not only broadcast its own information to all the other nodes within

r_u, but also relay such messages from other nodes. Given that the original topology is strongly connected, each node will eventually obtain the information on its visible neighborhood. This solution is, however, not localized.

Fortunately, the problem can still be solved locally for most cases. Consider a subgraph of G: $G' = (V(G'), E(G'))$, where $E(G') = \{(u, v) : d(u, v) \leq \min(r_u, r_v), u, v \in V(G)\}$. For any edge $(u, v) \in E(G')$, since $d(u, v) \leq \min(r_u, r_v)$, we have $(v, u) \in E(G')$, which means G' is bi-directional. Generally speaking, G' has fewer edges than G does, and may not be strongly connected even if G is strongly connected. Unless G is only "barely" connected, which seldom occurs, G' is usually strongly connected.

Define $N_u^{V'} = \{v \in V(G) : d(u, v) \leq \min(r_u, r_v)\}$, $r_u' = \max_{v \in N_u^{V'}}\{d(u, v)\}$, where $r_u' \leq r_u$ since for any $v \in N_u^{V'}$, $d(u, v) \leq r_u$. Let $r_{min}' = \min_{v \in V}\{r_v'\}$ and $r_{max}' = \max_{v \in V}\{r_v'\}$. By requiring each node u to broadcast its position and id to all other nodes within r_u, we are able to determine $N_u^{V'}$ and r_u'. We can then apply DRNG and DLSS to G' and prove that Theorem 4.5-Theorem 4.13 still hold if the original topology is G'.

Theorem 4.15. *Theorems 4.5, 4.8, 4.9, 4.12, 4.13, and 4.14 continue to hold if the original topology is* G'.

Proof. We replace G, r_u, N_u^V, r_{min}, and r_{max} with G', r_u', $N_u^{V'}$, r_{min}' and r_{max}' in the proof of Lemmas 4.4, 4.6, 4.7, 4.10 and Theorems 4.5, 4.8, 4.9, 4.12, 4.13, 4.14. By following the same line of arguments, we can prove that they still hold if the original topology is G'. □

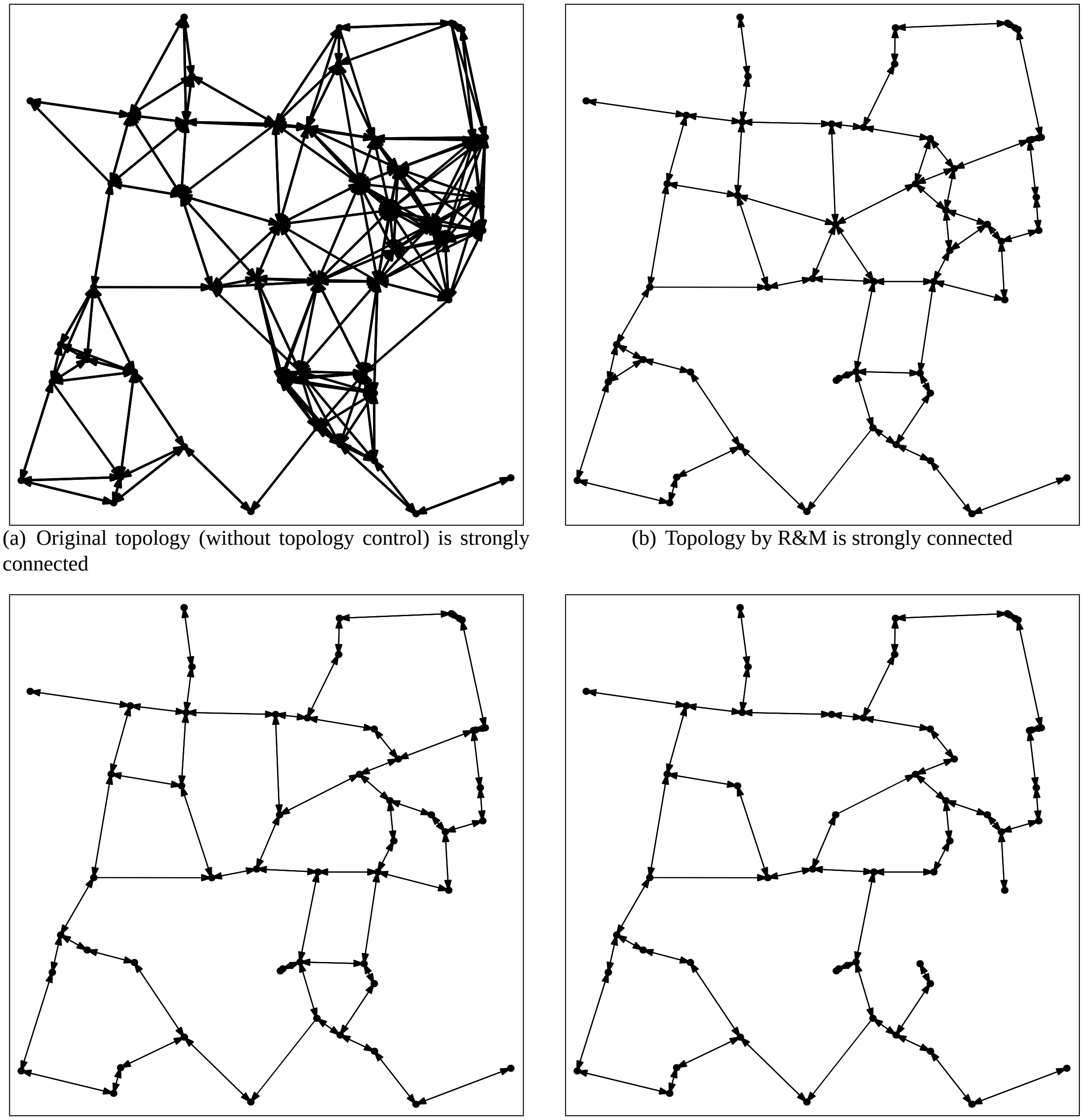
Theorem 4.16. *If the original topology is* G' *(which is a subgraph of* G*),* G_{DLSS} *and* G_{DRNG} *are bi-directional after* Addition *or* Removal.

Proof. We apply Theorem 4.9 to G', for G' is bi-directional. □

4.4 Performance Evaluation

In this section, we evaluate the performance of R&M, DRNG, and DLSS by simulations. All three algorithms are known to preserve network connectivity in heterogeneous networks. We do not compare with XTC [114] since XTC and DRNG share the similar idea (although they were from independent efforts).

In the first simulation, 50 nodes are uniformly distributed in a $1000 \times 1000m^2$ region. The transmission ranges of nodes are uniformly distributed in $[200m, 250m]$. Figure 4.11 gives the topologies derived using



(a) Original topology (without topology control) is strongly connected

(b) Topology by R&M is strongly connected

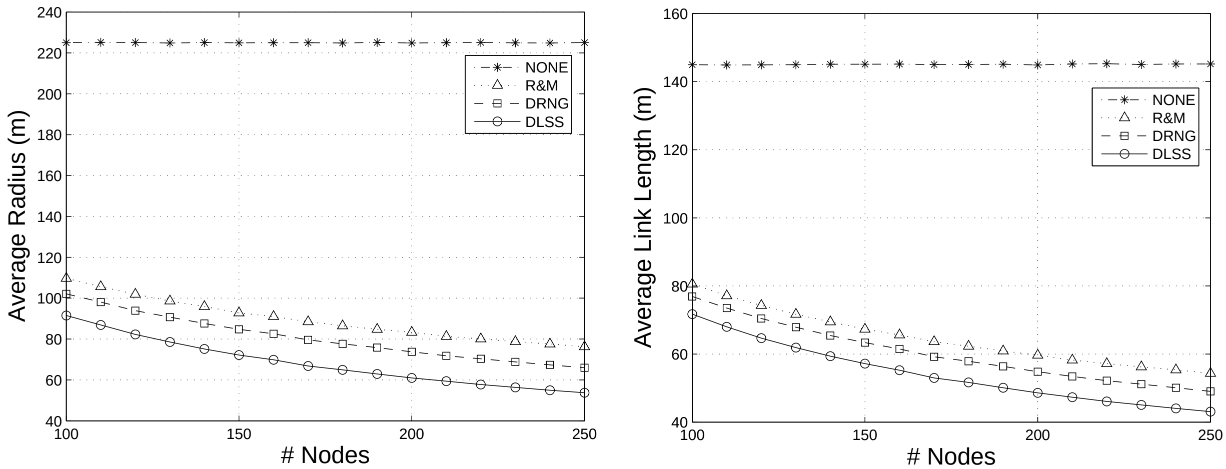
(c) Topology by DRNG is strongly connected

(d) Topology by DLSS is strongly connected

Figure 4.11: Network topologies by NONE, R&M, DRNG, and DLSS

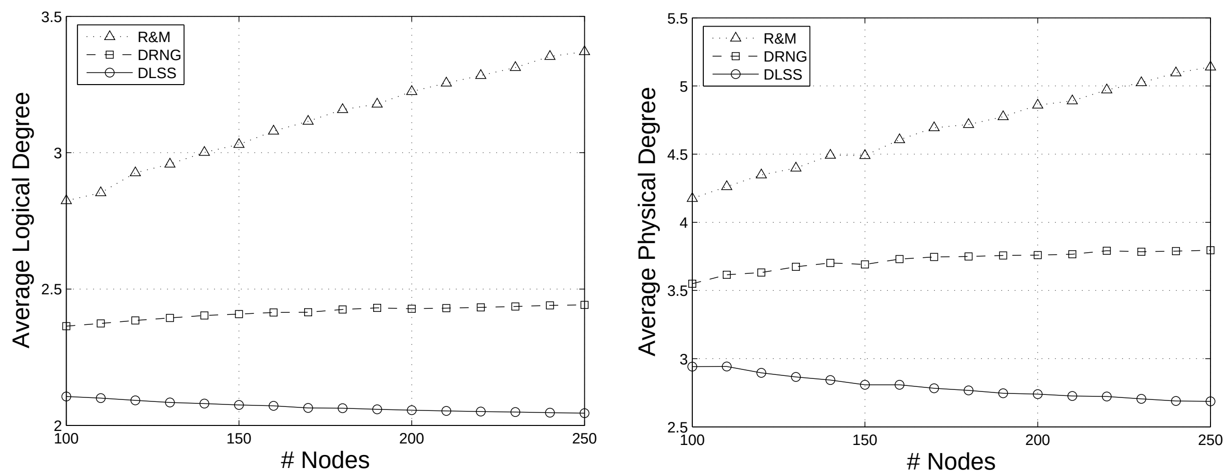
the maximal transmission power (labeled as NONE), R&M (under the Two-Ray Ground radion propagation model), DRNG, and DLSS for one simulation instance. As shown in Figure 4.11, R&M, DRNG and LMST all significantly reduce the average out-degree, while maintaining network connectivity. Moreover, there are fewer edges in DRNG and DLSS than in R&M.

In the second simulation, we vary the number of nodes in the region from 100 to 250. The transmission



(a) Average radius (b) Average link length

Figure 4.12: Performance comparison w.r.t. average radius and average link length

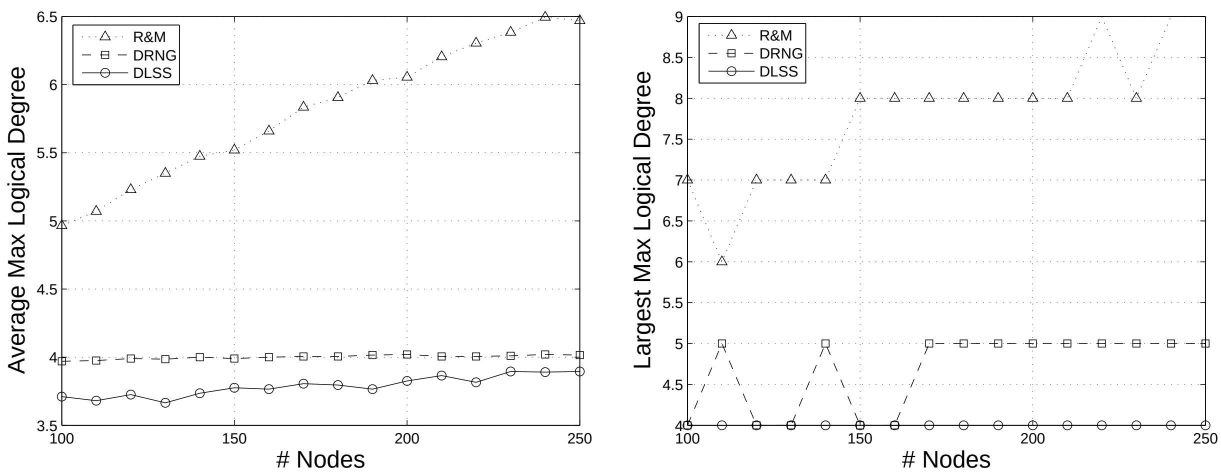


(a) Average logical out-degree (b) Average physical out-degree

Figure 4.13: Performance comparison w.r.t. average out-degree

ranges of nodes are uniformly distributed in $[200m, 250m]$. Each data point is the average of 200 simulation runs. Figure 4.12 shows the average radius and the average link length for the topologies generated by NONE (no topology control), R&M, DRNG, and DLSS. DLSS outperforms the others, which implies that DLSS can provide better spatial reuse and nodes in DLSS consume less energy to communicate with each other.

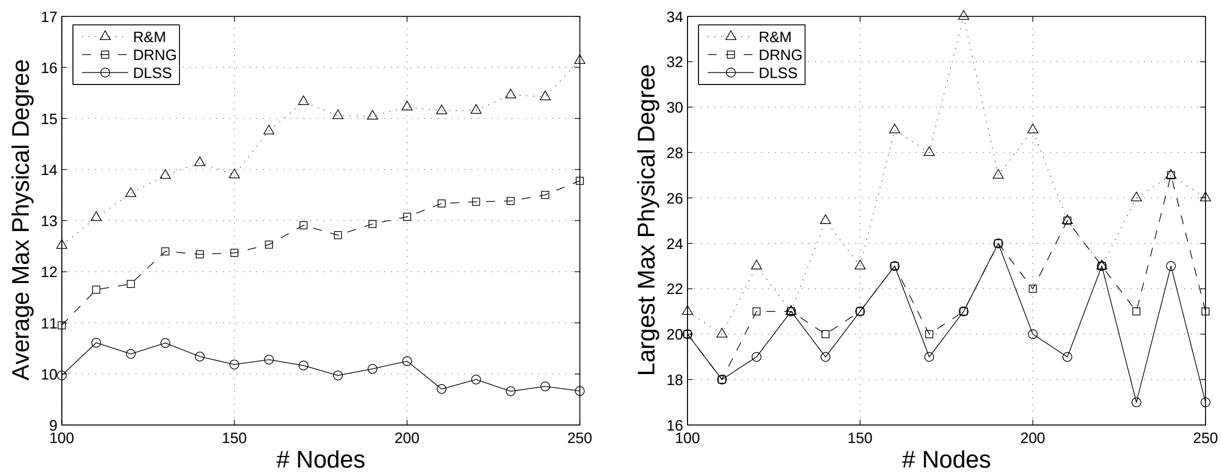
We also compare the out-degree of the topologies by different algorithms. The result of NONE is not shown because its out-degrees increase almost linearly with the number of nodes and are significantly larger than those under R&M, DRNG, and DLSS. Figure 4.13 shows the average logical/physical out-degree for



(a) Average maximum logical out-degree

(b) Largest maximum logical out-degree

Figure 4.14: Performance comparison w.r.t. maximum logical out-degree



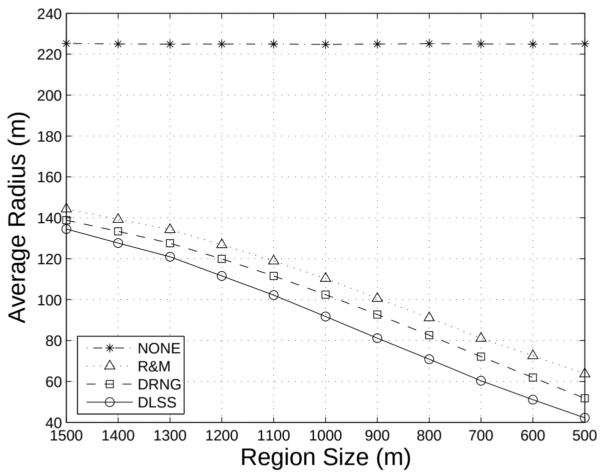
(a) Average maximum physical out-degree

(b) Largest maximum physical out-degree

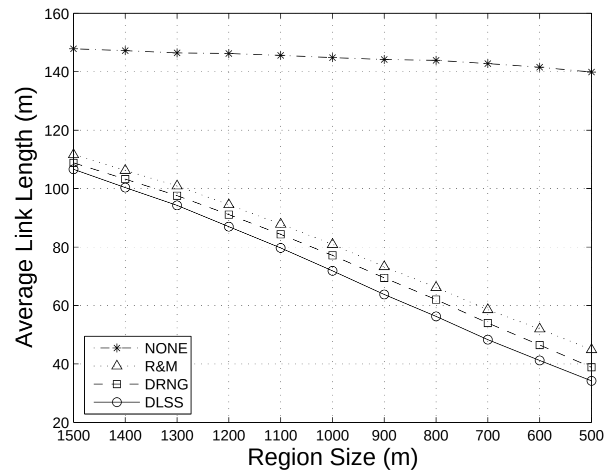
Figure 4.15: Performance comparison w.r.t. maximum physical out-degree

the topologies generated by R&M, DRNG, and DLSS. The average out-degrees under R&M and DRNG increase with the increase in the number of nodes, while that under DLSS actually decrease. The decrease in out-degree of DLSS can be explained as follows. With the increase of the density of nodes, the resulting topology of DLSS will resemble more and more closely that of a global directed minimum spanning tree (DMST). The average logical/physical out-degree of a global DMST is very small and stays relatively constant. As a result, the average degree of DLSS actually decreases with the increase of the node density.

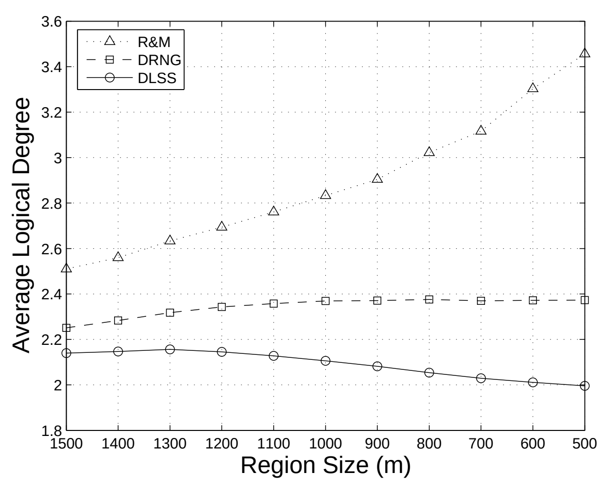
Figure 4.14 shows the average maximum logical degree and the largest maximum logical out-degree for the topologies generated by R&M, DRNG, and DLSS. The largest maximum logical out-degree under DLSS is at most 4, and is well below the theoretical upper bound obtained in Theorem 4.13. Also, the topology generated by DLSS has a much smaller average out-degree than the other topologies. Similar observations can be made in Figure 4.15 for the average physical degree. In addtion, the average physical degrees are always larger than the average logical degrees for the same topology. Note that the values in Figure 4.14 and Figure 4.15 do not monotonically increase or decrease with the increase of the node density, especially for the largest maximum physical degree of DLSS (Figure 4.15(b)). The reason is that those maximum values are largely determined by the node placement rather than the node density.



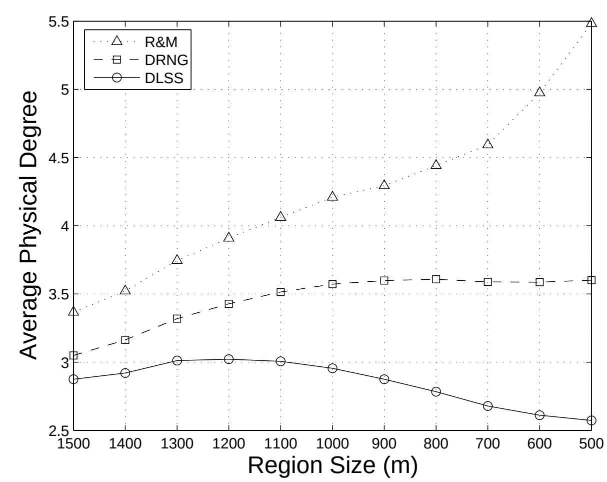
(a) Average radius



(b) Average link length



(c) Average logical out-degree



(d) Average physical out-degree

Figure 4.16: Comparison of R&M, DRNG and DLSS under different region sizes

In the third simulation, we fix the number of nodes in the network to be 100 and vary the size of the region from $1500 \times 1500m^2$ to $500 \times 500m^2$. The transmission ranges of nodes are uniformly distributed in $[200m, 250m]$ and each data point is the average of 200 simulation runs. Figure 4.16 shows the average radius, the average link length, and the average logical/physical out-degree. Similarly, we observe that DLSS outperforms other algorithms.

For homogeneous networks, the simulation results in Section 3.3 showed that LMST can increase the network capacity and improve the energy efficiency. For scenarios with relatively heavy traffic load, LMST can also deliver the packet more quickly since the wireless channel is shared in a more efficient manner. We believe DRNG and DLSS will perform similarly in heterogeneous networks as LMST does in homogeneous network.

4.5 Conclusions

We proposed two localized topology control algorithms, Directed Relative Neighborhood Graph (DRNG) and Directed Local Spanning Subgraph (DLSS), for heterogeneous wireless ad hoc networks in which each node may have different maximal transmission range. We show that most existing topology control algorithms (except [96, 114]) may generate disconnected network topologies when directly applied to heterogeneous networks. We proved that (1) both DRNG and DLSS preserve network connectivity; (2) both DRNG and DLSS preserve network bi-directionality; and (3) the out-degree of any node is bounded in the topology generated by DRNG or DLSS. The simulation study demonstrates that DRNG and DLSS outperform existing algorithms.

Chapter 5

Localized Fault-tolerant Topology Control

In this chapter, we discuss the issue of fault tolerance in topology control for wireless ad hoc networks. By using smallest possible transmission power that is connectivity-preserving, topology control algorithms tend to decrease the number of links, which in turn reduces the number of possible routing paths in the network. The resulting network topology is more likely to suffer from unpredictable events such as node failures. To design fault-tolerant topology control algorithms, we consider the k-connectivity of the network. A k-vertex connected network is $k-1$ fault-tolerant, i.e., the failure of at most $k-1$ nodes will not disconnect the network

For now we assume the network is homogeneous, i.e., the network topology G is undirected. We first propose a centralized greedy algorithm, *Fault-tolerant Global Spanning Subgraph* (FGSS$_k$) in Section 5.1, that preserves k-vertex connectivity and is min-max optimal, i.e., it minimizes the maximum transmission power used by nodes in the network. As will be discussed later, the min-max optimality is critical to prolong the network lifetime. Based on the centralized algorithm, we then propose a fully localized algorithm, *Fault-tolerant Local Spanning Subgraph* (FLSS$_k$). We prove that FLSS$_k$ preserves k-vertex connectivity and maintains bi-directionality for all the links in the topology, and is min-max optimal among all strictly localized algorithms. Finally, we further examine several widely used assumptions in topology control (e.g., homogeneous network, obstacle-free communication channel, and capability of obtaining position information) and discuss how to relax these assumptions.

Simulation study indicates that compared with the topologies generated by other distributed/localized fault-tolerant algorithms, the topology generated by FLSS$_k$ has a smaller average out-degree and smaller average transmission power. FLSS$_k$ also improves network capacity and energy efficiency. With modest adjustment, FLSS$_k$ can preserve the connectivity in spite of position estimation errors.

The rest of this chapter is organized as follows. We first present FGSS$_k$ in Section 5.1 and FLSS$_k$ in Section 5.2. Following that, we discuss in Section 5.3 how to relax several assumptions widely used in topology control so as to improve the practicality of FGSS$_k$ and FLSS$_k$. Finally, we present a simulation study in Section 5.4 and conclude this chapter in Section 5.5.

5.1 FGSS$_k$: Fault-tolerant Global Spanning Subgraph

In this section, we first present a centralized greedy algorithm, FGSS$_k$, that builds k-connected spanning subgraphs. FGSS$_k$ is a generalized version of Kruskal's algorithm for $k \geq 2$, where Kruskal's algorithm [57] is a well-known algorithm for constructing the minimum spanning tree (1-connected spanning subgraph) of a given graph. The FGSS$_k$ algorithm is described in Figure 5.1.

Procedure: FGSS$_k$
Input: $G(V, E)$, a k-connected simple graph;
Output: $G_k(V_k, E_k)$, a k-connected spanning subgraph of G;
begin
  $V_k := V$, $E_k := \emptyset$;
  Sort all edges in E in ascending order of weight
  for each edge (u_0, v_0) in the order
    if u_0 is not k-connected to v_0 in G_k
      $E_k := E_k \cup \{(u_0, v_0)\}$;
    endif
  end
end

Figure 5.1: FGSS$_k$ algorithm

Although FGSS$_k$ and FLSS$_k$ bear some similarity to CONNECT and BICONN-AUGMENT [94] in the way the topology is derived (i.e., different components are merged iteratively), they differ from the latter in that (1) FGSS$_k$ is more general, i.e., FGSS$_k$ preserves k-connectivity, while BICONN-AUGMENT only preserves 2-connectivity; (2) the correctness of BICONN-AUGMENT is only mentioned but not formally proved in [94], while a formal treatment of the correctness of FGSS$_k$ is given in this paper; (3) CONNECT and BICONN-AUGMENT are both centralized algorithms that require collection and distribution of global information, while FLSS$_k$ is fully localized; and (4) CONNECT and BICONN-AUGMENT operate under the assumption of homogeneous networks, while FGSS$_k$ and FLSS$_k$ can be (as will be formally proved in Section 5.3) applied to heterogeneous networks where the maximal transmission power of each node may be different.

By using network flow techniques [30], a query on whether two vertices are k-connected can be answered in $O(m\sqrt{n})$ time, where n is the number of vertices and m is the number of edges in the graph. Therefore, the time complexity of FGSS_k is $O(m^2\sqrt{n})$.

Although FGSS_k is a generalized version of Kruskal's algorithm, the techniques for the proof of correctness are completely different. Before proving the correctness of FGSS_k, we first provide two lemmas, which are also crucial to the proofs in Section 5.2.

Let the path from node u to node v in G be represented by a set p of vertices on the path, i.e., $p = \{u, w_1, w_2, \ldots, w_l, v\}$. Let $S_{uv}(F)$ be a maximal set of pairwise-vertex-disjoint paths from u to v in a graph F. Thus for $\forall p_1, p_2 \in S_{uv}(F)$, we have $p_1 \cap p_2 = \{u, v\}$. Let $F - (u_1, u_2)$ be the resulting graph by removing an edge (u_1, u_2) from F.

Lemma 5.1. *Let u_1 and u_2 be two vertices in a k-connected undirected graph F. If u_1 and u_2 are k-connected after the removal of edge (u_1, u_2), then $F - (u_1, u_2)$ is still k-connected.*

Proof. Equivalently, we prove that $F' = F - (u_1, u_2)$ is connected after the removal of any $k-1$ vertices from F'. Consider any two vertices v_1 and v_2 in F'. Without loss of generality, we assume $\{u_1, u_2\} \cap \{v_1, v_2\} = \emptyset$ (other cases can be proved using a similar approach). We now prove that v_1 is still connected to v_2 after removing a set of any other $k-1$ vertices $W = \{w_1, w_2, \ldots, w_{k-1}\}$, where $w_i \in V(F') - \{v_1, v_2\}$. This is obvious true if (v_1, v_2) is an edge in F. Therefore, we only consider the case where there is no edge from v_1 to v_2 in F. Since F is k-connected, $|S_{v_1v_2}(F)| \geq k$.

Let F'' be the resulting graph after (u_1, u_2) and W (and related edges) are removed from F, and let s_1 be the number of paths in $S_{v_1v_2}(F')$ that are broken due to the removal of vertices in W, i.e., $s_1 = |\{p \in S_{v_1v_2}(F') : \exists w \in W, w \in p\}|$. Since the paths in $S_{v_1v_2}(F')$ are pairwise-vertex-disjoint, the removal of any one vertex in W breaks at most one path in the set. Given $|W| = k-1$, we have $s_1 \leq k-1$.

If $|S_{v_1v_2}(F')| \geq k$, then $|S_{v_1v_2}(F'')| \geq |S_{v_1v_2}(F')| - s_1 \geq 1$, i.e., v_1 is still connected to v_2 in F''. Now we consider the case where $|S_{v_1v_2}(F')| < k$. This occurs only when the removal of (u_1, u_2) breaks one path $p^0 \in S_{v_1v_2}(F)$. Without loss of generality, let the order of vertices on the path be v_1, u_1, u_2, v_2. Since the removal of (u_1, u_2) reduces the number of pairwise-vertex-disjoint paths between v_1 and v_2 by at most one, $|S_{v_1v_2}(F) - \{p^0\}| \geq k-1$. Hence $|S_{v_1v_2}(F')| = k-1$. Now we consider two cases:

1. $s_1 < k-1$: $|S_{v_1v_2}(F'')| \geq |S_{v_1v_2}(F')| - s_1 \geq 1$, i.e., v_1 is still connected to v_2 in F''.

2. $s_1 = k-1$: hence every vertex in W belongs to some path in $S_{v_1v_2}(F')$. Since p^0 is internally-disjoint with all paths in $S_{v_1v_2}(F')$, we have $p^0 \cap W = \emptyset$. Thus v_1 is connected to u_1 and u_2 is connected to v_2 in F''. Let s_2 be the number of paths in $S_{u_1u_2}(F')$ that are broken due to the removal of vertices in W, i.e., $s_2 = |\{p \in S_{u_1u_2}(F') : \exists w \in W, w \in p\}|$. Since $|S_{u_1u_2}(F')| \geq k$ and $s_2 \leq k-1$, $|S_{u_1u_2}(F'')| \geq 1$, i.e., u_1 is still connected to u_2 in F''. Therefore, v_1 is still connected to v_2 in F''.

We have proved that for any two vertices $v_1, v_2 \in F'$, v_1 is connected to v_2 after the removal of any $k-1$ vertices from $F' - \{v_1, v_2\}$. Therefore, F' is k-connected. □

Lemma 5.2. *Let G and G' be two undirected simple graphs such that $V(G) = V(G')$. If G is k-connected, and every edge $(u,v) \in E(G) - E(G')$ satisfies that u is k-connected to v in $G - \{(u_0,v_0) \in E(G) : w(u_0,v_0) \geq w(u,v)\}$, then G' is also k-connected.*

Proof. Let $E = E(G) - E(G') = \{(u_1,v_1),(u_2,v_2),\ldots,(u_m,v_m)\}$ be a set of edges satisfying $w(u_1,v_1) \geq w(u_2,v_2) \geq \cdots \geq w(u_m,v_m)$. We define a series of graphs that are subgraphs of G. Let $G^0 = G$, and $G^i = G^{i-1} - (u_i,v_i)$, $i = 1,2,\ldots,m$. Now we prove by induction.

1. *Base*: $G^0 = G$ is k-connected.

2. *Induction*: If G^{i-1} is k-connected, we prove that G^i is k-connected, where $i = 1,2,\ldots,m$. Since $G - \{(u_0,v_0) \in E(G) : w(u_0,v_0) \geq w(u_i,v_i)\} \subseteq G^{i-1} - (u_i,v_i)$, u_i is k-connected to v_i in $G^{i-1} - (u_i,v_i)$. Applying Lemma 5.1 to G^{i-1}, we can prove that $G^i = G^{i-1} - (u_i,v_i)$ is still k-connected.

Now we have proved that G^m is k-connected. Since $E(G^m) \subseteq E(G')$, G' is also k-connected. □

Theorem 5.3. *FGSS$_k$ can preserve k-connectivity of G, i.e., G_k is k-connected if G is k-connected.*

Proof. Since edges are inserted into G_k in ascending order, whether u is k-connected to v at the moment before (u,v) is inserted depends only on the edges of smaller weight. Therefore, every edge $(u,v) \in E_0 = E(G) - E(G_k)$ satisfies that u is k-connected to v in $G - \{(u,v) \in E(G) : w(u,v) \geq w(u_0,v_0)\}$. we can prove that G_k preserves k-connectivity of G by applying Lemma 5.2 to G_k. □

Let $\rho(F)$ be the largest radius of all nodes in F, i.e., $\rho(F) = \max_{u \in V(F)}\{R_u\}$. Now we prove that FGSS$_k$ achieves the min-max optimality, i.e., let $SS_k(G)$ be the set of all k-connected spanning subgraphs of G, then $\rho(G_k) = \min\{\rho(F) : F \in SS_k(G)\}$. This optimality is proved in [94] for $k = 2$. Here we extend the result to arbitrary k.

Theorem 5.4. *The maximum transmission radius (or equivalently, power) among all nodes is minimized by FGSS$_k$, i.e.,* $\rho(G_k) = \min\{\rho(F) : F \in SS_k(G)\}$.

Proof. Suppose G is k-connected. By Theorem 5.3, G_k is also k-connected. Let (u, v) be the last edge that is inserted into G_k; we have $w(u, v) = \max_{(u_0,v_0)\in E(G_k)} \{w(u_0, v_0)\}$ and $R_u = \rho(G_k)$. Let $G'_k = G_k - (u, v)$, we have $|S_{uv}(G'_k)| < k$; otherwise according to the algorithm in Figure 5.1, (u, v) should not be included in G_k. Now consider a graph $H = (V(H), E(H))$, where $V(H) = V(G)$ and $E(H) = \{(u_0, v_0) \in E(G) : w(u_0, v_0) < w(u, v)\}$. If we can prove that H is not k-connected, we will be able to conclude that any $F \in SS_k(G)$ must have at least one edge equal to or longer than (u, v), which means $\rho(G_k) = \min\{\rho(F) : F \in SS_k(G)\}$.

Now we prove by contradiction that H is not k-connected. Assume H is k-connected and hence $|S_{uv}(H)| \geq k$. We have $E(H) \nsubseteq E(G'_k)$; otherwise, $|S_{uv}(G'_k)| \geq |S_{uv}(H)| \geq k$. Therefore, $E_0 = E(H) - E(G'_k) \neq \emptyset$. Since edges are inserted into G'_k in ascending order, $\forall(u_1, v_1) \in E_0$ satisfies that u_1 is k-connected to v_1 in $H - \{(u_0, v_0) \in E(H) : w(u_0, v_0) \geq w(u_1, v_1)\}$. By Lemma 5.2, we can prove that G'_k is k-connected. This means $|S_{uv}(G'_k)| \geq k$, which is a contradiction. □

The min-max optimality of FGSS$_k$ is an important feature. The network lifetime is usually defined as the time it takes for the first node to deplete its energy. Assume a static network where each node has the same initial energy. If the traffic pattern is random and each node forwards approximately equal amount of traffic, then the energy consumption of each node is roughly proportional to its transmission power. Therefore, the network lifetime is very likely to be determined by the node that uses the maximum tranmission power among all nodes. By minimizing the maximum transmission power, FGSS$_k$ achieves the maximum network lifetime.

5.2 FLSS$_k$: Fault-tolerant Local Spanning Subgraph

In this section, we present a localized algorithm, *Fault-tolerant Local Spanning Subgraph* (FLSS). FLSS is also composed of three steps: information collection, topology construction, and construction of topology with only bi-directional links. We will only explain the process of topology construction here, since the other two steps are similar to those in Section 3.1.

Given the visible neighborhood N_u^V, each node u builds its local spanning subgraph $S_u = (V(S_u), E(S_u))$ over N_u^V using the algorithm described in Figure 5.2.

Procedure: FLSS$_k$
Input: G_u^V, u's visible neighborhood;
Output: S_u, a spanning subgraph of G_u^V;
begin
    $V(S_u) := V(G_u^V)$, $E(S_u) := \emptyset$;
    Sort all edges in $E(G_u^V)$ in ascending order of weight
    for each edge (u_0, v_0) in the order
        if u_0 is not k-connected to v_0 in S_u
            $E(S_u) := E(S_u) \cup \{(u_0, v_0)\}$;
        endif
    end
end

Figure 5.2: FLSS$_k$ algorithm

Definition 5.5 (Neighbor Relation in FLSS$_k$)**.** *In Fault-tolerant Local Spanning Subgraph (FLSS$_k$), node v is a out-neighbor of node u, denoted $u \xrightarrow{FLSS} v$, if and only if $(u, v) \in E(S_u)$. That is, v is a neighbor of u if and only if v is an immediate neighbor on u's local spanning subgraph S_u.*

The network topology under FLSS$_k$ is all the nodes in $V(G)$ and their individually perceived neighbor relations. Note that the topology is *not* a simple superposition of all local spanning subgraphs. In addition, the neighbor relation defined above is not symmetric, i.e., $u \xrightarrow{FLSS} v$ does not necessarily imply $v \xrightarrow{FLSS} u$.

Theorem 5.6 (Connectivity of FLSS$_k$)**.** *If G is k-connected, then G_{FLSS}, G^+_{FLSS} and G^-_{FLSS} are all k-connected.*

Proof. We only need to prove that G^-_{FLSS} preserves k-connectivity of G, for $E(G^-_{FLSS}) \subseteq E(G_{FLSS}) \subseteq E(G^+_{FLSS})$. Since G^-_{FLSS} is bi-directional, we can treat it as an undirected graph. Let $E = E(G) - E(G^-_{FLSS})$. For any edge $e = (u, v) \in E$, at least one of (u, v) and (v, u) was not in G_{FLSS}, since $e \notin E(G^-_{FLSS})$. Without loss of generality assume (u, v) was not in G_{FLSS}. Thus in the process of local topology construction of node u, u was already k-connected to v before (u, v) was inspected. Since edges are inserted in ascending order, whether u is k-connected to v at the moment before (u, v) is inspected depends only on the edges of smaller weights. Therefore, u is k-connected to v in $G - \{(u_0, v_0) \in E(G) : w(u_0, v_0) > w(u, v)\}$. Let $G' = G^-_{FLSS}$, we can conclude that G^-_{FLSS} is k-connected by Lemma 5.2. $\square$

Definition 5.7 (Strictly Localized Algorithms)**.** *An algorithm is strictly localized if its operation on any node u is based only on the information that is originated from the nodes in N_u^V.*

For any node u running a strictly localized algorithm, the information that u may rely on is quite limited. For instance, suppose that u and $v \in N_u^V$ is not k-connected in G_u^V. Since it is impossible for u to know whether it is k-connected to v in G, u has to keep the local "connectedness" as much as possible. In other words, if u and v are not k-connected before edge (u, v) is examined, u has to keep v as its out-neighbor in the final topology.

Let $LSS_k(G)$ be the set of all k-connected spanning subgraphs of G that are constructed by strictly localized algorithms. Now we prove that FLSS achieves the min-max optimality among all strictly localized algorithms, i.e., $\rho(G_{FLSS}) = \min\{\rho(F) : F \in LSS_k(G)\}$.

Theorem 5.8. *Among all strictly localized algorithms, FLSS$_k$ minimizes the maximum transmission radius (or power) of nodes in the network, i.e., $\rho(G_{FLSS}) = \min\{\rho(F) : F \in LSS_k(G)\}$.*

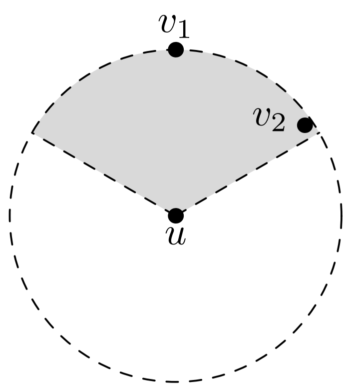
Proof. Suppose G is k-connected. Let (u, v) be the last edge inserted into G_{FLSS}. We have $w(u, v) = \max_{(u_0,v_0)\in E(G_{FLSS})} \{w(u_0, v_0)\}$ and $R_u = \rho(G_{FLSS})$. Let G_0 be the induced subgraph of G_{FLSS} where $V(G_0) = N_u^V$, and let $G'_0 = G_0 - \{(u, v)\}$. We have $|S_{uv}(G'_0)| < k$; otherwise (u, v) should not be included in G_0. Also define $H_0 = (V(H_0), E(H_0))$, where $V(H_0) = V(G_u^V)$ and $E(H_0) = \{(u_0, v_0) \in E(G_u^V) : w(u_0, v_0) < w(u, v)\}$.

To prove that H_0 is not k-connected, we replace G, G_k, G'_k, and H with G_u^V, G_0, G'_0, and H_0 respectively, and follow the corresponding proof in Theorem 5.4. After proving H_0 is not k-connected, we consider the following cases:

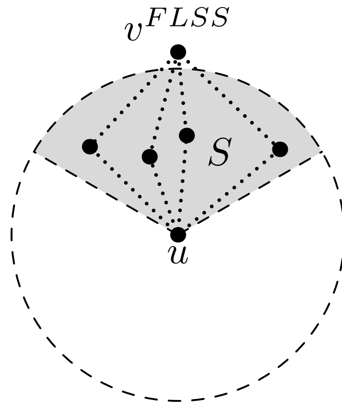
1. u is k-connected to v in G_u^V: since H_0 is not k-connected, any $F \in LSS_k(G)$ should have had at least one edge equal to or longer than (u, v),

2. u is not k-connected to v in G_u^V: to preserve the connectedness as much as possible, any $F \in LSS_k(G)$ should have included (u, v);

In both cases, $\rho(F) \geq \rho(G_u^V) = \rho(G_{FLSS})$, which means $\rho(G_{FLSS}) = \min\{\rho(F) : F \in LSS_k(G)\}$. $\square$



(a) Lemma 5.9



(b) Theorem 5.10 and Theorem 5.11

Figure 5.3: Illustrations for Theorem 5.10 and Theorem 5.11

Let R_u^{FLSS}, R_u^{CBTC}, and R_u^{YAO} be the radius of any node u under FLSS$_k$, CBTC($\frac{2\pi}{3k}$) [4], and Yao$_{p,k}$ [75], respectively. We derive the relationship of R_u^{FLSS}, R_u^{CBTC}, and R_u^{YAO}.

Lemma 5.9. *For three nodes $u, v_1, v_2 \in G$, if $\angle v_1 u v_2 \leq \pi/3$ and $w(u, v_2) < w(u, v_1)$, then $w(v_1, v_2) < w(u, v_1)$.*

Proof. If $w(u, v_2) < w(u, v_1)$, then $d(u, v_2) \leq d(u, v_1)$. We have $d(v_1, v_2) \leq d(u, v_1)$, since $\angle v_1 u v_2 \leq \pi/3$. Consider the following two cases (Figure 5.3(a)):

1. $d(v_1, v_2) < d(u, v_1)$. It is obvious that $w(v_1, v_2) < w(u, v_1)$.

2. $d(v_1, v_2) = d(u, v_1)$. This only occurs when $d(v_1, v_2) = d(u, v_1) = d(u, v_2)$. Since $w(u, v_2) < w(u, v_1)$, u, v_1 and v_2 satisfy one of the three scenarios (out of six possible scenarios): $id(u) < id(v_2) < id(v_1)$, $id(v_2) < id(u) < id(v_1)$, and $id(v_2) < id(v_1) < id(u)$. We can check for all three scenarios, $w(u, v_2) < w(u, v_1)$.

□

Theorem 5.10. *$R_u^{FLSS} \leq R_u^{CBTC}$ for any node $u \in G$.*

Proof. We prove by contradiction. Let v^{FLSS} be the farthest out-neighbor of u in G_{FLSS}. Suppose $R_u^{FLSS} = d(u, v^{FLSS}) > R_u^{CBTC}$. Recall that there are two stages in CBTC. In the first stage, u increases its power until the maximum angle between two consecutive out-neighbors is at most $\frac{2\pi}{3k}$. If the cone coverage cannot be satisfied when u is already transmitting with the maximal power, u will attempt to reduce its power subject to maintaining the cone coverage in the second stage.

Consider the number of out-neighbors of u in the shaded area S in Figure 5.3(b)(which is an open, minor sector with a radius of R_u^{CBTC} and an angle of $\frac{2\pi}{3}$ at the center). If the CBTC algorithm stops in the first stage, u has at least $\frac{2\pi}{3}/\frac{2\pi}{3k} = k$ nodes in S in order to fulfill the coverage requirement. If the algorithm proceeds to the second stage, u has at least k out-neighbors in S in order to shrink back and maintain k-connectivity at the same time [4]. In both cases, u has at least k out-neighbors in S. Let those nodes be $v_i, i = 1, 2, \ldots, L$, where $L \geq k$.

Now consider the local topology construction process of u in FLSS. Since $d(u, v_i) < d(u, v^{FLSS}) \leq r_{max}$ and $d(v_i, v^{FLSS}) < d(u, v^{FLSS}) \leq r_{max}$ (by Lemma 5.9), $i = 1, 2, \ldots, L$, there are at least k pairwise-vertex-disjoint paths from u to v^{FLSS} in G (i.e., $(u, v_i, v^{FLSS}, i = 1, 2, \ldots, L \geq k)$). As we have proved that FLSS does not affect the k-connectivity between u and v^{FLSS}, u and v^{FLSS} are already k-connected when edge (u, v^{FLSS}) is examined in FLSS. Therefore, v^{FLSS} cannot be a out-neighbor of u in G_{FLSS}, which contradicts with our assumption. □

Theorem 5.11. $R_u^{FLSS} \leq R_u^{YAO}$ *for any node* $u \in G$.

Proof. We also prove by contradiction. Let v^{FLSS} be the farthest out-neighbor of u in G_{FLSS}. Suppose $R_u^{FLSS} = d(u, v^{FLSS}) > R_u^{YAO}$. Recall that in Yao$_{p,k}$, each node u increase it power until it has k closest out-neighbors in each of $p \geq 6$ equal cones around u. Since $R_u^{YAO} < d(u, v^{FLSS})$, u has at least k out-neighbors that are closer to itself than v^{FLSS} in the cone where v^{FLSS} resides. Given that $p \geq 6$, there are at least k out-neighbors of u in S (Figure 5.3(b)). Using an argument similar to that in Theorem 5.10, we can conclude that v^{FLSS} cannot be a out-neighbor of u in G_{FLSS}, which contradicts with our assumption. □

Theorem 5.10 and Theorem 5.11 show that FLSS$_k$ outperforms both CBTC($\frac{2\pi}{3k}$) and Yao$_{p,k}$ in terms of the transmission radius, which is also corroborated by the simulation study in Section 5.4. While both CBTC($\frac{2\pi}{3k}$) and YAO$_{p,k}$ impose certain coverage constraints in each individual cone, FLSS$_k$ only imposes requirements on the entire visible neighborhood. Take a node u in CBTC($\frac{2\pi}{3k}$) for example, if we assume a relative high density of nodes, u has to have at least $2\pi/(\frac{2\pi}{3k}) = 3k$ out-neighbors, w.h.p., to ensure that the maximum angle between any two consecutive out-neighbors is at most $\frac{2\pi}{3k}$. This bound is actually not very tight since most likely it will take more out-neighbors to fulfill the requirement of cone coverage. Similarly, the out-degree of a node for YAO$_{p,k}$ is at least pk (k nodes in each of the p cones), w.h.p.

5.3 Practical Considerations

Although the assumptions stated in Section 2.2 are widely used in existing topology control algorithms, some of them are made to facilitate algorithm construction and analysis, and may not be practical. In this section, we discuss how to relax these assumptions for FGSS$_k$ and FLSS$_k$ as well as algorithm previously proposed in Chapter 3 and Chapter 4, so as to improve their practicality in real applications.

5.3.1 Relaxing Assumption of Homogeneous Network

As mentioned in [69], the assumption of homogeneous nodes does not always hold in practice, since even devices of the same type may have slightly different maximal transmission power, let alone the fact that devices of different types possess dramatically different capabilities. Fortunately FGSS$_k$/FLSS$_k$ can be readily applied in heterogeneous networks.

Now we prove that FGSS$_k$ preserves k-connectivity and is min-max optimal even in heterogeneous networks. The following results correspond to Lemma 5.1, Lemma 5.2, Theorem 5.3 and Theorem 5.4, respectively. The proof is literally the same as that in Section 5.1, except that now we consider directed graphs consisting of directed edges. This resemblance is by no means a coincidence, since we actually consider more general cases when we proved all the theorems and lemmas in Section 5.1 and Section 5.2.

Lemma 5.1*. *Let u_1 and u_2 be two vertices in a k-connected* directed *graph F. If u_1 is k-connected to u_2 after the removal of edge (u_1, u_2), then $F - (u_1, u_2)$ is still k-connected.*

Lemma 5.2*. *Let G and G' be two* directed *simple graphs such that $V(G) = V(G')$. If G is k-connected, and every edge $(u, v) \in E(G) - E(G')$ satisfies that u is k-connected to v in $G - \{(u_0, v_0) \in E(G) : w(u_0, v_0) \geq w(u, v)\}$, then G' is also k-connected.*

Theorem 5.3*. *FGSS$_k$ can preserve k-connectivity in heterogeneous networks, i.e., G_k is k-connected if G is k-connected, where G is a directed graph.*

Theorem 5.4*. *The maximum transmission radius (or power) among all nodes is minimized by FGSS$_k$, i.e., $\rho(G_k) = \min\{\rho(F) : F \in SS_k(G)\}$, where G is a directed graph and $SS_k(G)$ is the set of all k-connected spanning subgraphs of G.*

The following theorem proves that FLSS$_k$ preserves k-connectivity. Note that G^-_{FLSS} can no longer preserve k-connectivity for heterogeneous networks.

Theorem 5.6* (Connectivity of FLSS$_k$). *If G is k-connected, then G_{FLSS} and G^+_{FLSS} are both k-connected.*

Proof. We only need to prove that G_{FLSS} preserves k-connectivity of G, since $E(G_{FLSS}) \subseteq E(G^+_{FLSS})$. Let $E = E(G) - E(G_{FLSS})$. For any edge $e = (u, v) \in E$, it is not in $E(G_{FLSS})$ because in the process of local topology construction of node u, u was already k-connected to v before (u, v) was inserted. Since edges are inserted in an ascending order, whether u is k-connected to v at the moment before (u, v) is inserted depends only on the edges of smaller weights. Therefore, u is k-connected to v in $G - \{(u_0, v_0) \in E(G) : w(u_0, v_0) > w(u, v)\}$. Let $G' = G_{FLSS}$, we conclude that G_{FLSS} is k-connected by Lemma 2*. □

The min-max optimality of FLSS$_k$ can be proved in a straightforward manner:

Theorem 5.8*. *Among all strictly localized algorithms, FLSS$_k$ minimizes the maximum transmission radius (or power) of nodes in the network, i.e., $\rho(G_{FLSS}) = \min\{\rho(F) : F \in LSS_k(G)\}$, where G is a directed graph.*

5.3.2 Relaxing Assumption of Obstacle-free Communication Channel

We assumed in Section 2.2 an obstacle-free communication channel, which is not always applicable, especially for indoor communications [105]. In this subsection, we argue that this assumption can be readily dismissed.

We have previously assumed that the original network topology, G, is a general directed or undirected graph. The information needed by FGSS$_k$ and FLSS$_k$ is the edges that exist in G. An edge between nodes u and v is not formed in the network, either because u and v are not within the transmission range of each other, or because there exist obstacles in between. No matter what is the reason, non-existent edges are not considered in the construction of topologies, in FGSS$_k$ and FLSS$_k$. As long as the original topology (which has taken into consideration of the effect of obstacles) is k-connected, FGSS$_k$ and FLSS$_k$ can be applied to provide a min-max optimal solution to preserve the k-connectivity. Therefore, the assumption of an obstacle-free wireless channel can be relaxed without any modification to FLSS$_k$.

5.3.3 Relaxing Requirement on Position Information

It is assumed in Section 2.2 that each node is equipped with the capability to obtain its own location information. In this subsection, we discuss how to relax this assumption.

As mentioned in Section 5.3.2, what is required by FGSS_k and FLSS_k is the information of all the existing edges in the network. In order to obtain such information, each node u includes in its Hello message its node id and position. With all the messages that reach a node v, node v can then infer the local topology in the visible neighborhood N_v^V with the position information.

Note that our algorithms can still operate if the position information is unavailable, as only the knowledge of all the existing edges, $E(G^V)$, is required. $E(G^V)$ can be constructed locally as follows. First, each node periodically broadcasts, using its maximal transmission power, a very short Hi message which includes only its node *id* and its maximal transmission power. Upon receiving such a message from a neighbor node v, each node u estimates the length of the edge (u, v) based on the attenuation incurred in the transmission. Let the set of edges incident at u be denoted as $E_u^T = \{(u, v) : v \in N_u^V\})$. After node u collects the information on E_u^T, it can then broadcast this information in an Edge message. Each node will be able to infer $E(G_u^V)$ based on the Edge messages received from all of its neighbors. Although this solution may incur more communication and computation overhead, and make our algorithms less "localized", it eliminates the need for the position information.

5.3.4 Relaxing Assumption of Perfect Omni-directional Antennas

Many topology control algorithms assume that the antenna pattern of a wireless device is a perfect disk. This is also the underlying assumption for algorithms that use explicit channel propagation models. Since the same models are applied to all directions, the antenna pattern has to be isotropic, which in turn implies that the area covered by a transmission is a perfect disk.

In the case of FGSS/FLSS, the antenna pattern model influences the manner in which the information of N_u^V can be collected. Given an arbitrary antenna pattern, we can simply employ the information dissemination technique in the previous section. It is obvious that the information dissemination technique does not rely on any specific antenna pattern. The edge length, however, is no longer a good measure of weight, since the power attenuation may vary in different directions.

Instead, we use $Pl(u, v)$, the minimal transmission power for u to reach v, as the weight of an edge

(u, v) (along with nodes' *id*s as tie-breakers). Each node u broadcasts periodically a Hello message that includes the *id* and the maximal transmission power. Upon receiving such a message from a neighbor node u, node v calculates $Pl(u, v)$ based on the received signal strength. Once each node obtains the information of all edges in its visible neighborhood, DLSS can be applied to construct the topology.

5.4 Performance Evaluation

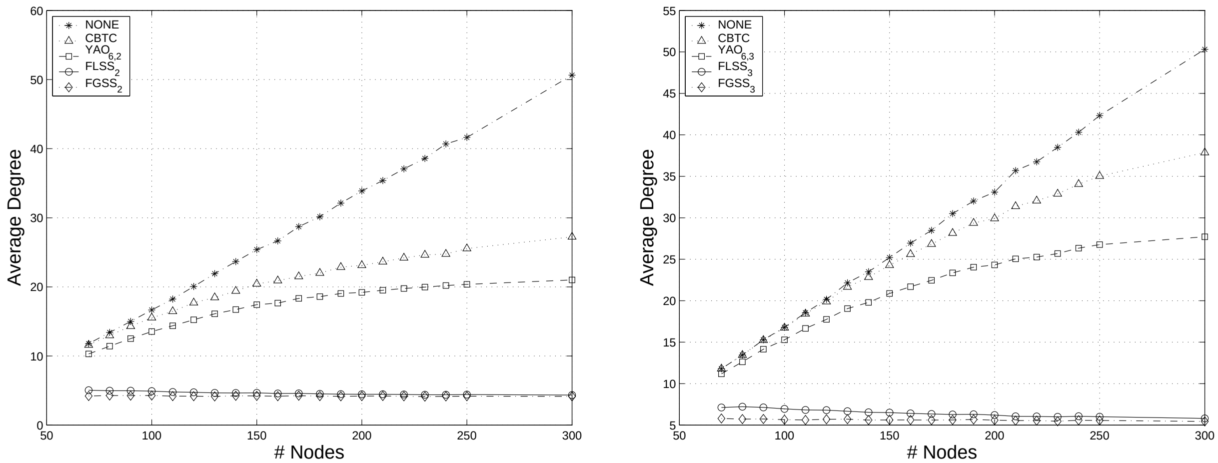
In this section, we evaluate the performance of FLSS$_k$ by comparing with CBTC($\frac{2\pi}{3k}$) [4], Yao$_{p,k}$ [75], and Hajiaghayi's algorithms [40], with respect to several metrics via simulations. We set $p = 6$ in Yao$_{p,k}$ in order to minimize the average power [75]. For the sake of fair comparison, we have to use several assumptions that are common to all algorithms, e.g., the Unit Disk Graph (UDG) model. The performance of the centralized algorithm FGSS$_k$ is also shown as a baseline. As will be shown in the following discussions, the performance of FLSS$_k$ is only slightly worse than that of FGSS$_k$.

5.4.1 Comparison between CBTC($\frac{2\pi}{3k}$), Yao$_{p,k}$, and FLSS$_k$

In the first set of simulations, we study the performance with respect to out-degree, maximum radius, and energy saving. Nodes are uniformly distributed in a $1000 \times 1000m^2$ region. The transmission range of every node is $261.195m$, which corresponds to a transmission power of 0.28183815 watt under the Free Space propagation model. We vary the number of nodes in the region from 70 to 300. Each data point is the average of 50 simulation runs.

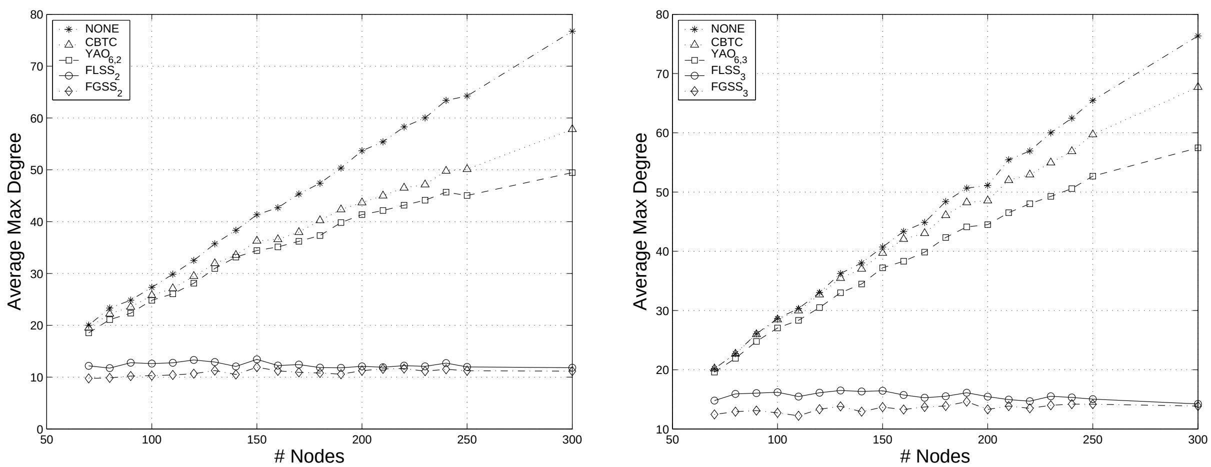
Node Degree We compare the average physical out-degree of the topologies generated by different algorithms. Recall that the physical out-degree is defined as the number of nodes within the transmission radius of a node. The out-degree is a good indication of the level of possible MAC interference (and hence the extent of spatial reuse), i.e., the smaller the out-degree of a node, the less number of nodes its transmissions may interfere with, and potentially affect.

Figure 5.4 shows the average out-degree of the topologies generated by CBTC($\frac{2\pi}{3k}$), YAO$_{6,k}$, FLSS$_k$ and FGSS$_k$, for $k = 2$ and $k = 3$. The average out-degree under NONE (with no topology control) increases almost linearly with the number of nodes. The average out-degree under CBTC($\frac{2\pi}{3k}$) and YAO$_{6,k}$ also increases as the number of nodes increases. In contrast, the average out-degree under FGSS$_k$ and FLSS$_k$



(a) $k = 2$ (b) $k = 3$

Figure 5.4: Performance comparison w.r.t. average physical out-degree

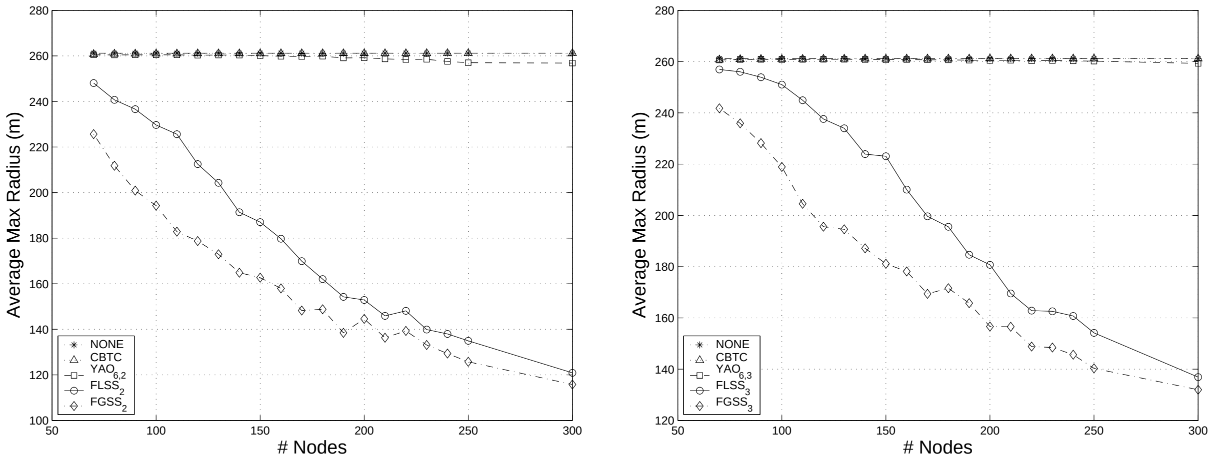


(a) $k = 2$ (b) $k = 3$

Figure 5.5: Performance comparison w.r.t. average maximum out-degree

actually slightly decreases. The average out-degrees of both CBTC($\frac{2\pi}{3k}$) and YAO$_{6,k}$ are much higher than that of FLSS$_k$ since in FLSS$_k$, nodes always has a smaller transmission radius (as proved in Section 5.2).

Figure 5.5 gives the average maximum out-degree in the topologies generated by CBTC($\frac{2\pi}{3k}$), YAO$_{6,k}$, FLSS$_k$, and FGSS$_k$, for $k = 2$ and $k = 3$. The average maximum out-degree under FGSS$_k$ and FLSS$_k$ is significantly smaller than that under NONE/CBTC($\frac{2\pi}{3k}$)/YAO$_{6,k}$. All results show that FLSS$_2$ can achieve better spatial reuse, and the performance improvement becomes even more prominent when the network density becomes higher.



(a) $k = 2$ (b) $k = 3$

Figure 5.6: Performance comparison w.r.t. average maximum radius

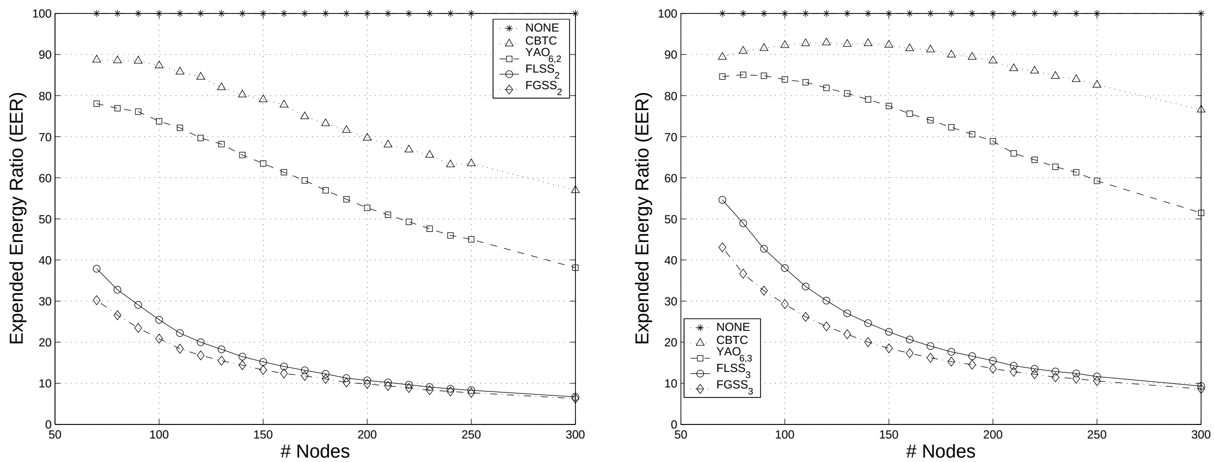
Maximum Radius The average maximum radius for the topologies under CBTC($\frac{2\pi}{3k}$), $\text{YAO}_{6,k}$, FLSS_k and FGSS_k are shown in Figure 5.6, for $k = 2$ and $k = 3$. The average maximum radius of CBTC($\frac{2\pi}{3k}$) or $\text{YAO}_{6,k}$ comes very close to that of NONE, which implies that CBTC($\frac{2\pi}{3k}$) and $\text{YAO}_{6,k}$ cannot really prolong the network lifetime (given the lifetime defined in Section 5.1). In contrast, the average maximum radius of FLSS_k is significantly smaller. Moreover, its performance is very close to that of the centralized algorithm FGSS_k.

Energy Saving We compare the various algorithms with respect to the average *Expended Energy Ratio* (EER), where EER is defined in [40] as

$$EER = \frac{E_{ave}}{E_{max}} \times 100,$$

E_{ave} is the average transmission power over all the nodes in the network, and E_{max} is the maximal transmission power that can reach the transmission range of $261.195m$. Here we use the free-space propagation model to calculate the transmission power. Figure 5.7 gives the comparison results for both $k = 2$ and $k = 3$. FLSS_k clearly has the advantage. The intuition behind this is that the average out-degree of FGSS/FLSS is much smaller.

Network Capacity and Energy Efficiency In the second set of simulations, we compare CBTC($\frac{2\pi}{3k}$), $\text{YAO}_{6,k}$, FLSS_k and FGSS_k with respect to network capacity and energy efficiency using the *ns-2* simulator



(a) $k = 2$ (b) $k = 3$

Figure 5.7: Performance comparison w.r.t. EER

[79]. In this set of simulations, n nodes are randomly distributed in a $150 \times 20m^2$ region, with half of them being sources and the other half being destinations. We use a rectangular region for the same reasons described in Section 3.3.2.

In the simulation, the MAC protocol is IEEE 802.11, the routing protocol is AODV, and the traffic sources are CBR and TCP traffic with bulk FTP sources. The start time of each connection is chosen randomly from $[0s, 10s]$. Each simulation run lasts for 100 seconds, and each data point in the figures is the average of 30 simulation runs.

The *Log-Normal Shadowing Model* is used to characterize the near-ground communications in an open field [31]. Let α be the path loss exponent, d be the distance between the transmitter and the receiver, and d_0 be a short distance. If $P_r(d)$ is the received power at the receiver, $P_r(d_0)$ is the received power at a distance d_0 from the transmitter, then the path loss in dB, $P_L(d)$, is given by

$$P_L(d) = 10 \log \left[\frac{P_r(d_0)}{P_r(d)} \right] = 10\alpha \log(\frac{d}{d_0}) + X_{dB}, \tag{5.1}$$

where X_{dB} is a Gaussian random variable that has a zero mean and a standard deviation σ_{dB}. $P_r(d_0)$ can be calculated by using the Free Space Model (Equation 3.1). According to the measurement in [31], here let $\alpha = 3.41$ and $\sigma_{dB} = 4.70$.

The performance metrics are:

- Total data delivery (end-to-end) in the network (in bytes).

- Total energy consumption in the network (in Joules).
- Throughput energy efficiency (in bytes/Joule): the total data delivered (in bytes) divided by the total energy consumption (in Joules).
- Network capacity (in byte·meters): the sum of products of bits and the distances over which they are carried [37] (also known as the transport capacity).
- Transport energy efficiency (in byte·meters/Joule): the total transport capacity (in byte·meters) divided by the total energy consumption (in Joules).
- Total receptions: total number of successful receptions in the network. It is a good indicator of spatial reuse.

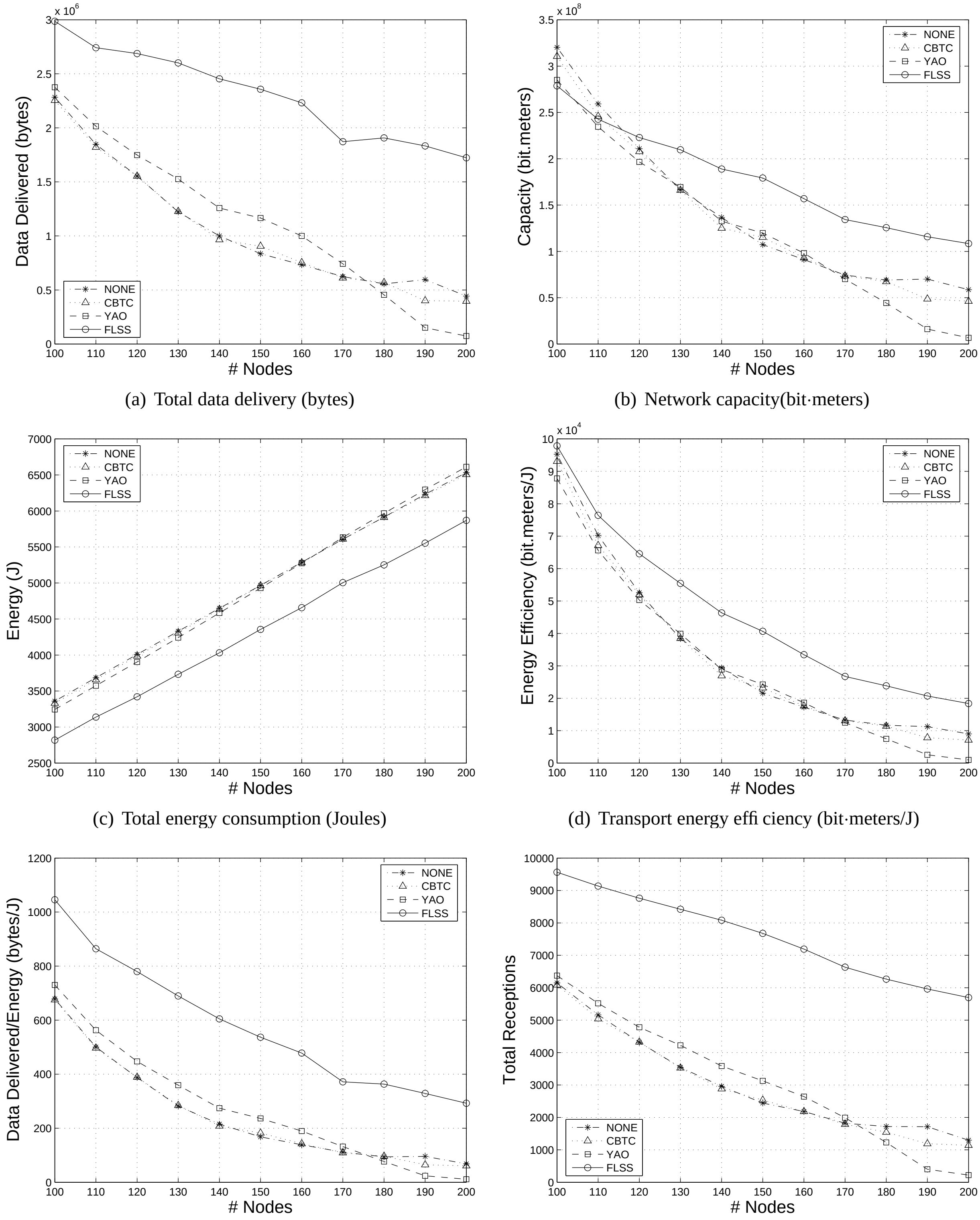
For CBR traffic, Figure 5.8 and Figure 5.9 compare the network capacity and the energy efficiency for $k = 2$ and $k = 3$, respectively. On one hand, the difference between the results obtained under CBTC/YAO and those under no topology control is not significant. On the other hand, FLSS_k not only improves the network capacity significantly, but also is the most energy-efficient. The results under TCP/FTP traffic exhibit the similar trends (shown in Figure 5.10 and Figure 5.11).

5.4.2 Comparison between k-UPVCS and FGSS_k/FLSS_k

In the third set of simulations, we compare FGSS_k and FLSS_k with both the distributed and the centralized versions of k-UPVCS [40] in terms of EER, for both $k = 2$ and $k = 3$. The simulation is conducted in a similar setting to that in [40]. As we are unable to accurately control the density of the original graph (with the maximal transmission power), we compare the algorithms under the topology of roughly the same average out-degree. Figure 5.12 gives the comparison results. FLSS performs better than the distributed version of k-UPVCS in almost every setting, though FGSS performs worse than the global version of k-UPVCS. The latter is probably due to the fact that FGSS is simply a greedy algorithm.

5.4.3 Trade-off between Topology Robustness and Performance

In the fourth set of simulations, we compare FLSS_1 (i.e., LMST [71]), FLSS_2 and FLSS_3. As shown in Figure 5.13, as k increases, FLSS_k renders topologies that have larger average out-degrees, longer average



(a) Total data delivery (bytes)

(b) Network capacity(bit·meters)

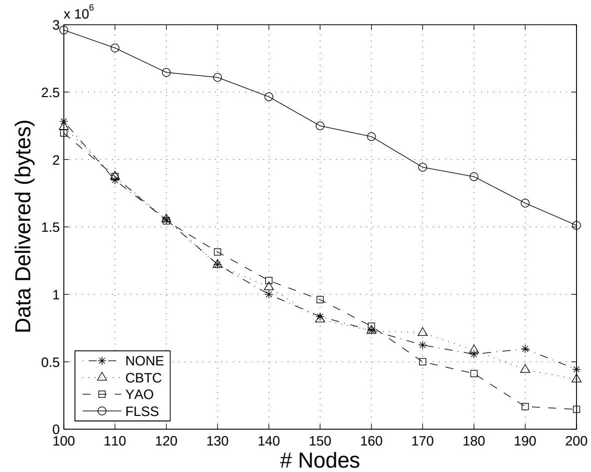
(c) Total energy consumption (Joules)

(d) Transport energy effi ciency (bit·meters/J)

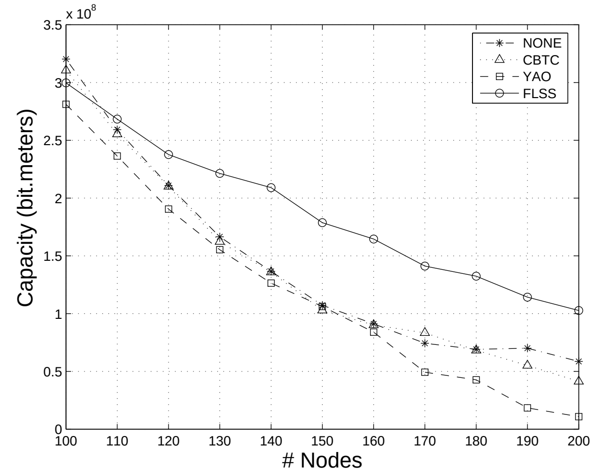
(e) Throughput energy effi ciency (bytes/J)

(f) Total receptions

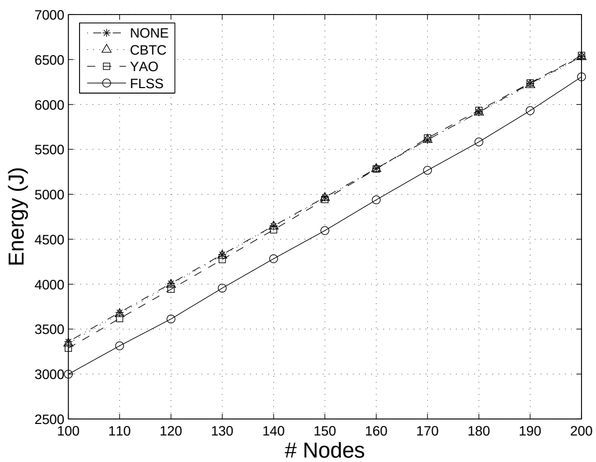
Figure 5.8: Network capacity and energy efficiency under CBR traffic ($k = 2$)



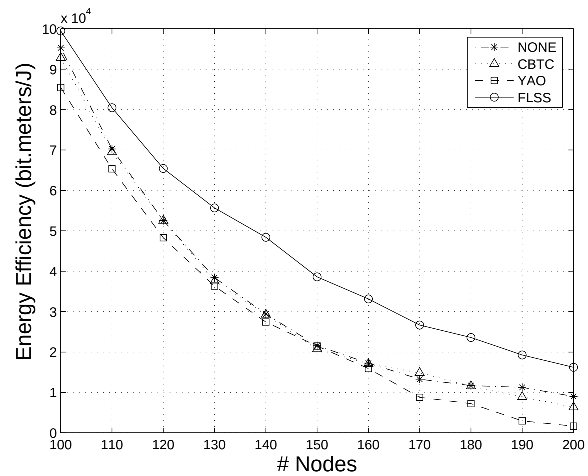
(a) Total data delivery (bytes)



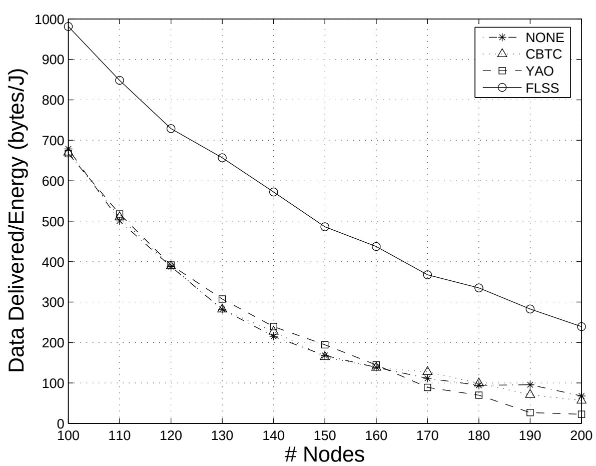
(b) Network capacity(bit·meters)



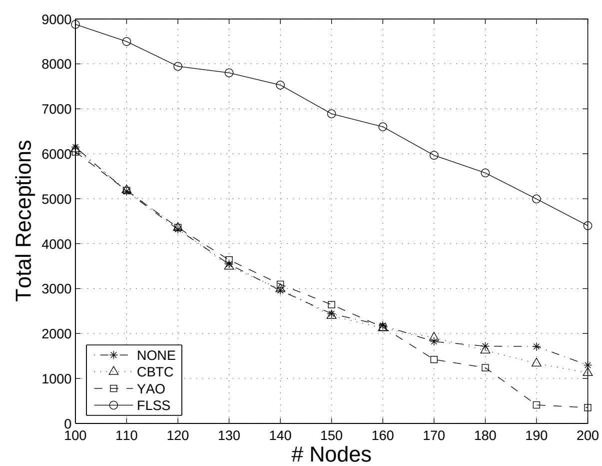
(c) Total energy consumption (Joules)



(d) Transport energy effi ciency (bit·meters/J)

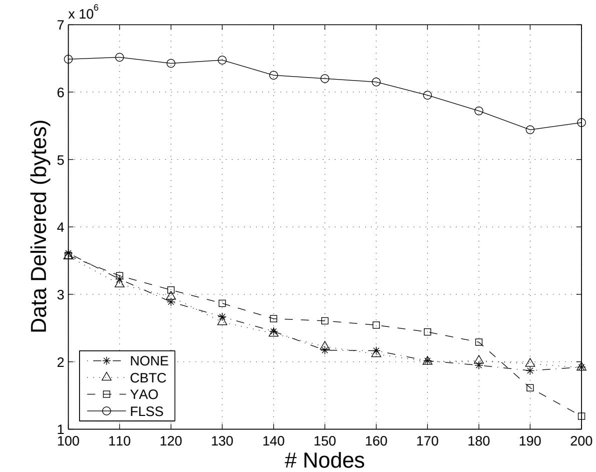


(e) Throughput energy effi ciency (bytes/J)

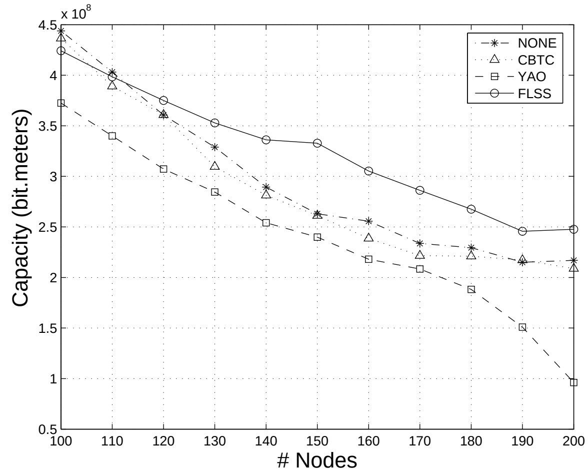


(f) Total receptions

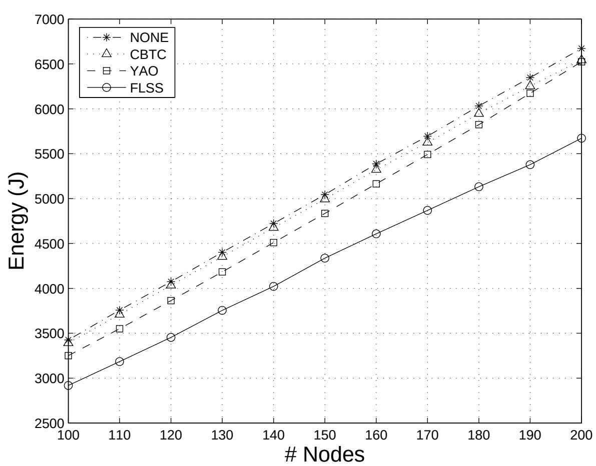
Figure 5.9: Network capacity and energy efficiency under CBR traffic ($k = 3$)



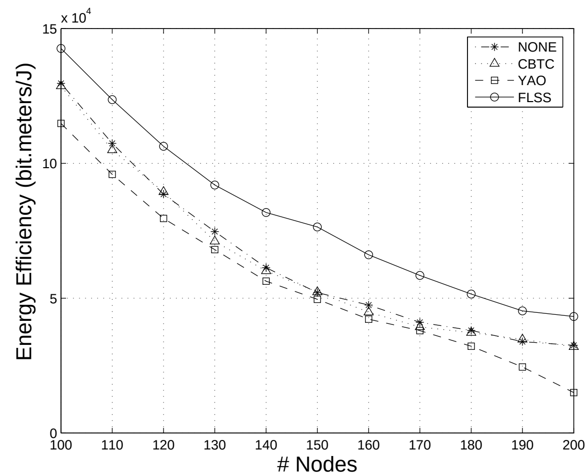
(a) Total data delivery (bytes)



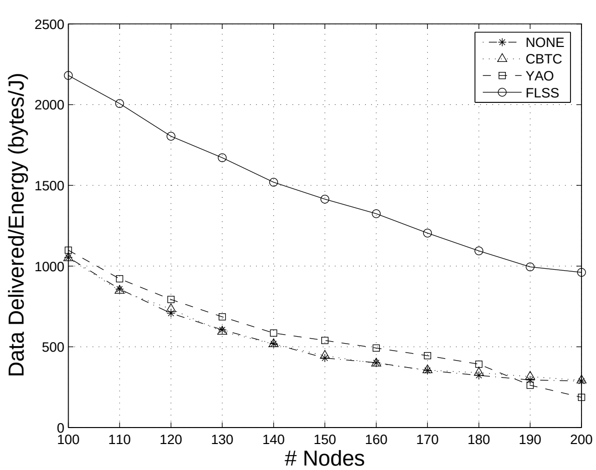
(b) Network capacity(bit·meters)



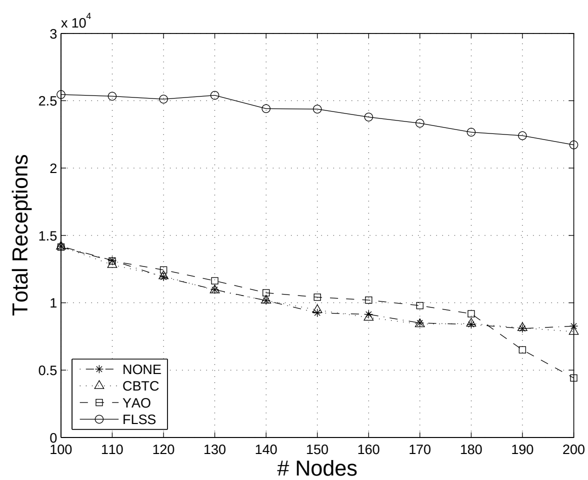
(c) Total energy consumption (Joules)



(d) Transport energy effi ciency (bit·meters/J)

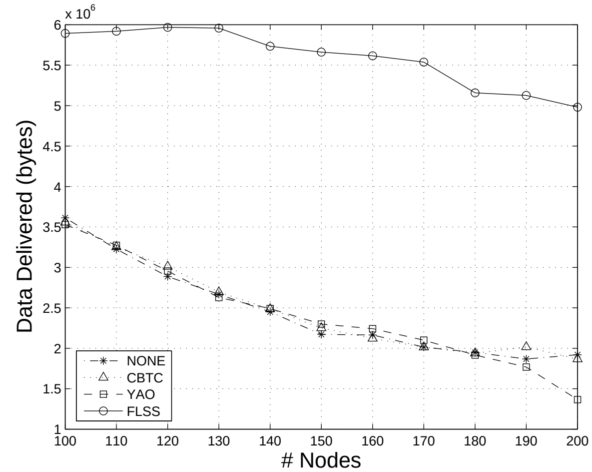


(e) Throughput energy effi ciency (bytes/J)

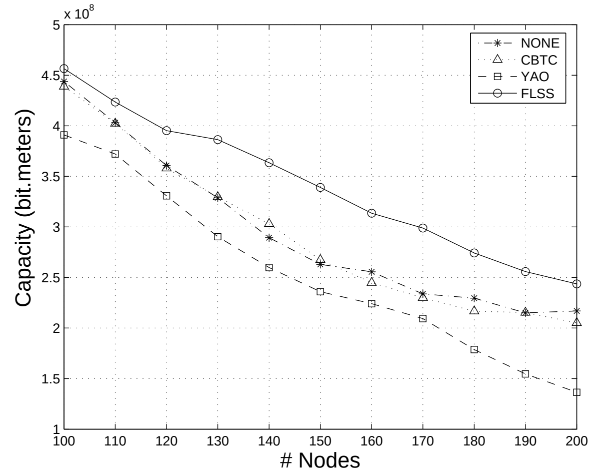


(f) Total receptions

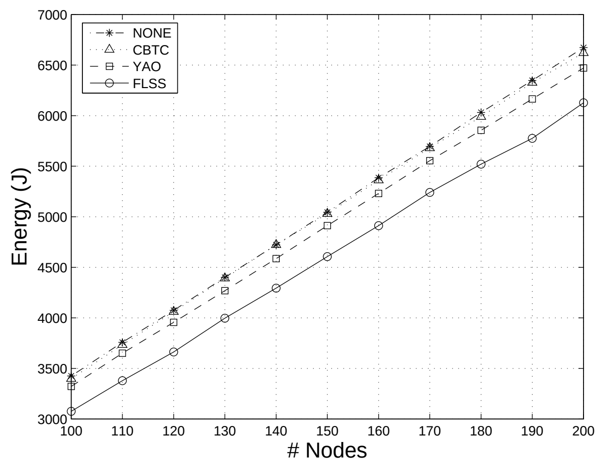
Figure 5.10: Network capacity and energy efficiency under TCP/FTP traffic ($k = 2$)



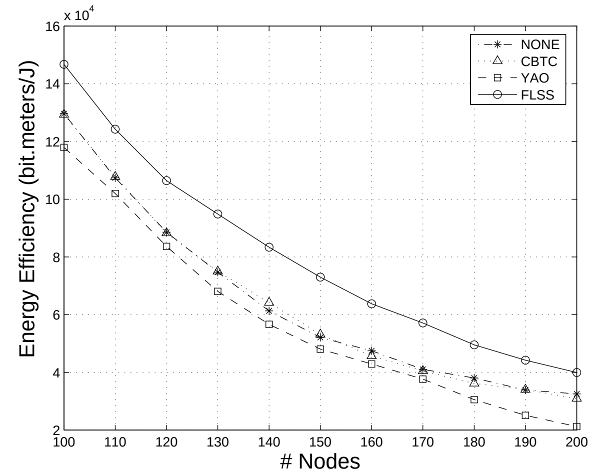
(a) Total data delivery (bytes)



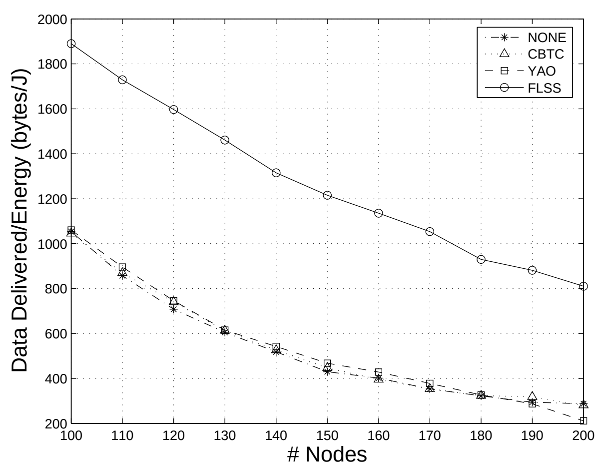
(b) Network capacity(bit·meters)



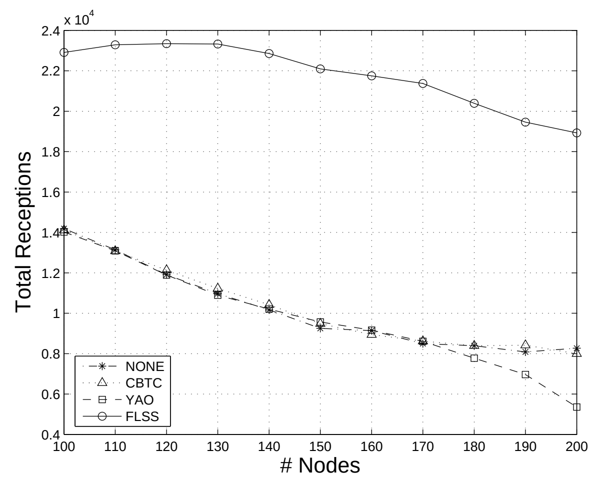
(c) Total energy consumption (Joules)



(d) Transport energy effi ciency (bit·meters/J)

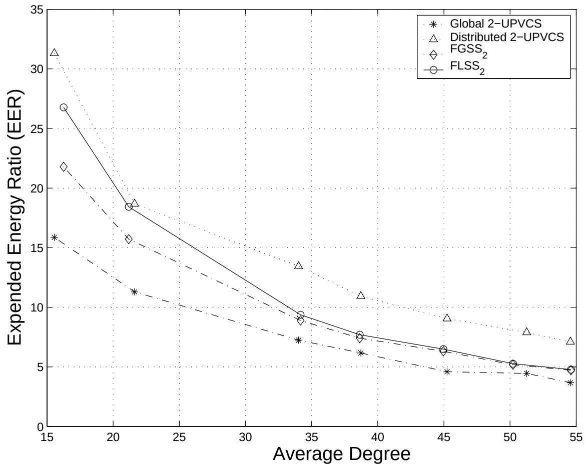


(e) Throughput energy effi ciency (bytes/J)

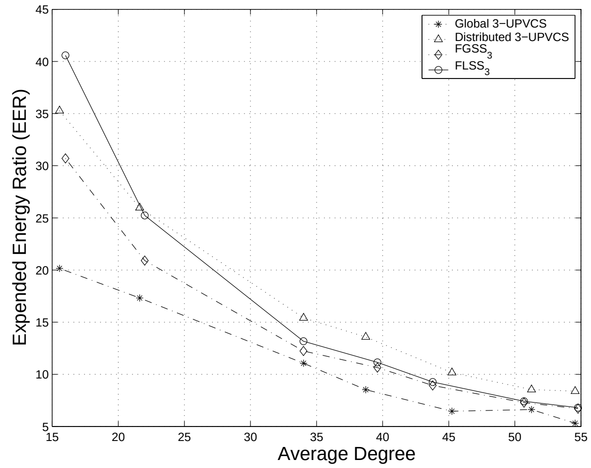


(f) Total receptions

Figure 5.11: Network capacity and energy efficiency under TCP/FTP traffic ($k = 3$)

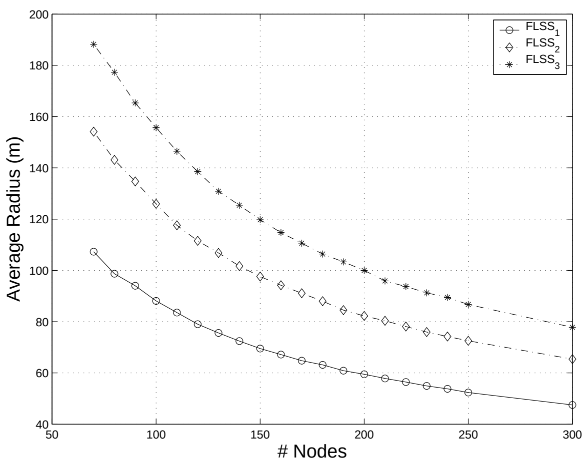


(a) $k = 2$

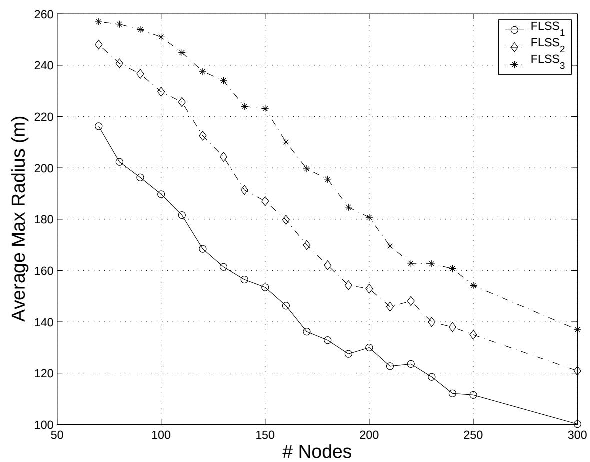


(b) $k = 3$

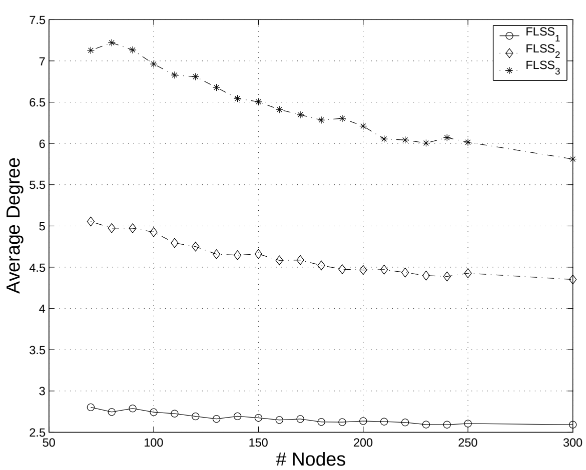
Figure 5.12: Comparison of FGSS/FLSS and k-UPVCS w.r.t. EER



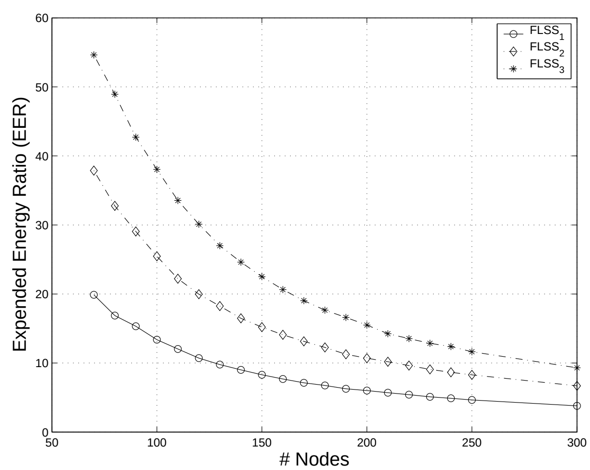
(a) Average radius



(b) Average maximum radius

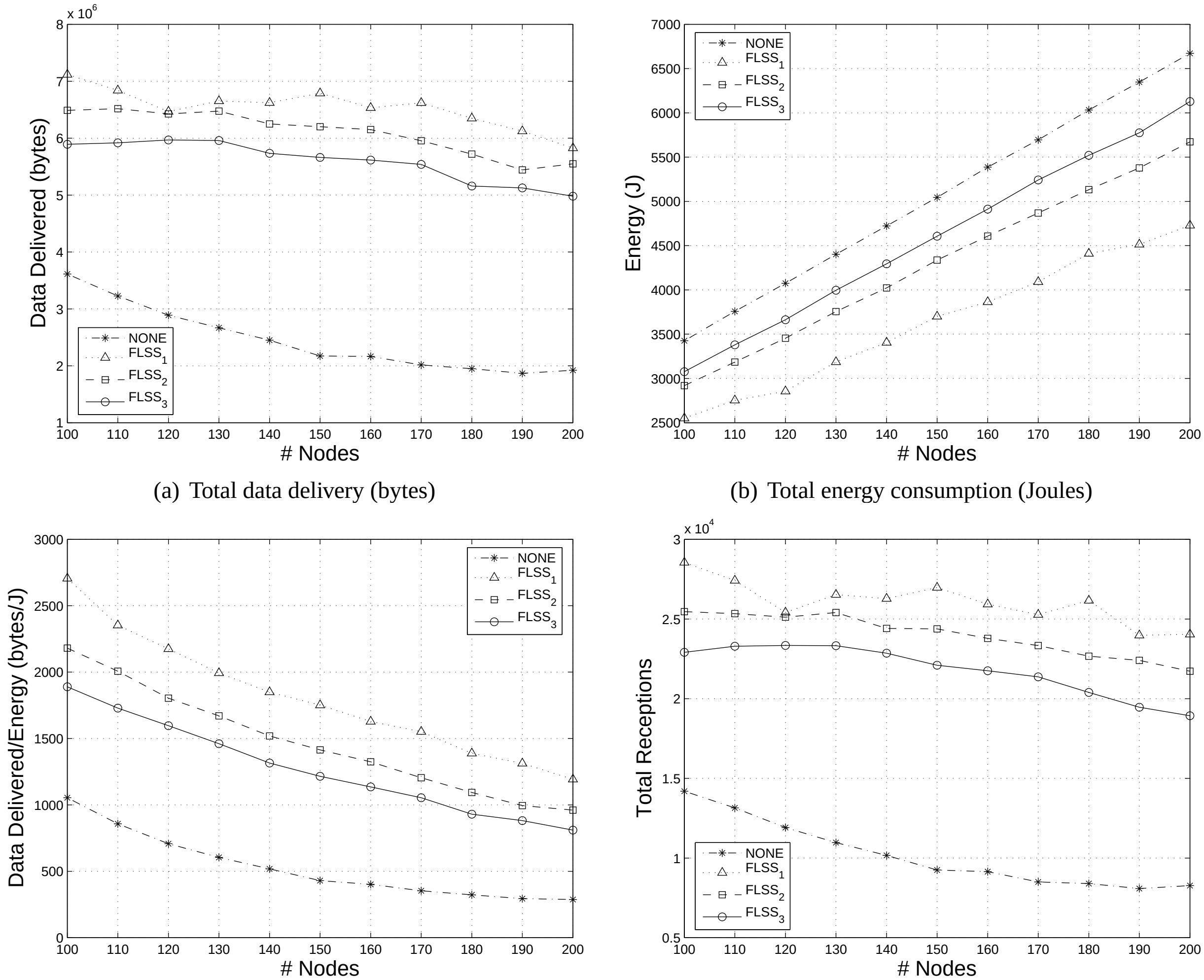


(c) Average physical out-degree



(d) EER

Figure 5.13: Comparison of $FLSS_k$ (k=1,2,3) w.r.t. radius, out-degree and EER



(a) Total data delivery (bytes)

(b) Total energy consumption (Joules)

(c) Throughput energy effi ciency (bytes/J)

(d) Total receptions

Figure 5.14: Comparison of FLSS$_k$ (k=1,2,3) w.r.t. network capacity and energy efficiency (TCP traffic)

radii, and longer average maximum radii, and consume more power. However, the topologies are also more robust and are resilient to $k-1$ failures. This shows the trade-off between the robustness of the topology and the other performance metrics (e.g., power consumption, network lifetime, spatial reuse, and MAC Layer interference).

We also compare FLSS$_1$, FLSS$_2$ and FLSS$_3$ with respect to network capacity and energy efficiency. The simulation settings are the same as those in the second set of simulations. Figure 5.14 shows results for TCP/FTP traffic. It can be observed that with the increase in the level of network connectivity (in the order of FLSS$_1$, FLSS$_2$, FLSS$_3$, NONE), the total throughput decreases, the total energy consumption increases, and the energy efficiency decreases. This result again demonstrates the trade-off between the robustness (or routing redundancy) and the network capacity/energy efficiency.

5.4.4 Robustness w.r.t. Position Estimation Errors

In Section 2.2, we assumed that each node can estimate its own position by using various localization techniques. In Section 5.3.3, we relaxed this assumption by having each node estimate the length of edges it is incident to. Both methods will introduce errors on the position estimation. As a result, FLSS may not be able to preserve k-connectivity. In the next set of simulations, we investigate how robust FLSS is, with respect to the position estimation error.

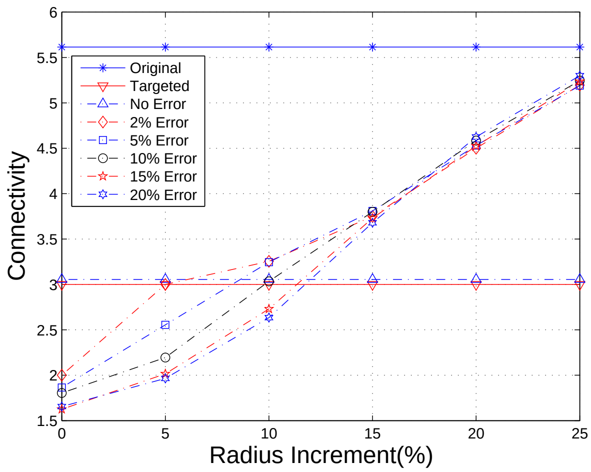
Suppose the real position of a node u is (u_x, u_y), and the estimated position of u is modeled to be randomly distributed inside the disk centered at (u_x, u_y). Given the error rate E_r, the radius of the disk is $E_r \cdot r_{max}$ (where r_{max} is the maximal transmission range). For a given targeted connectivity, FLSS is used to build the network topology based on the estimated positions. To deal with the errors incurred in the estimation, we increase the transmission radius of each node, in the hope that the network connectivity can be repaired. The increase is proportional to the transmission radius.

For this set of simulation, nodes are randomly distributed in a square region of $2r_{max}$ by $2r_{max}$, where $r_{max} = 261.195m$. The number of nodes N varies from 30 to 60, and the targeted connectivity K varies from 3 to 6. The error rate varies from 2% to 20%. Thus, the maximum error varies from $2\% \cdot r_{max}$ ($\sim 5.2m$) to $20\% \cdot r_{max}$ ($\sim 52m$). The increase in the transmission radius varies from 0% to 25%. Each data point is the average of 200 simulation runs.

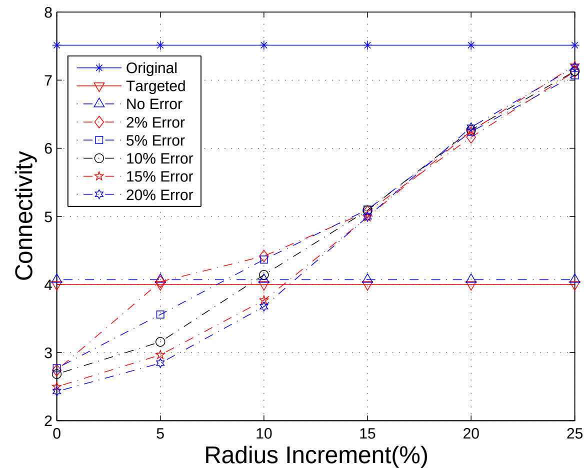
Figure 5.15 gives the average connectivity under different error rates on positions and different increases in the transmission radius. The estimation error does influence the preservation of connectivity in FLSS. An modest increase in the transmission radius, however, will boost the connectivity to the desired level. We further look into the standard deviation of the connectivity for $N = 30$ and $N = 60$ in Figure 5.16 and Figure 5.17, respectively. Generally, if each node increases its transmission radius by 15%, the targeted connectivity can be achieved in spite of the position estimation errors, which shows that FLSS is quite robust.

5.5 Conclusions

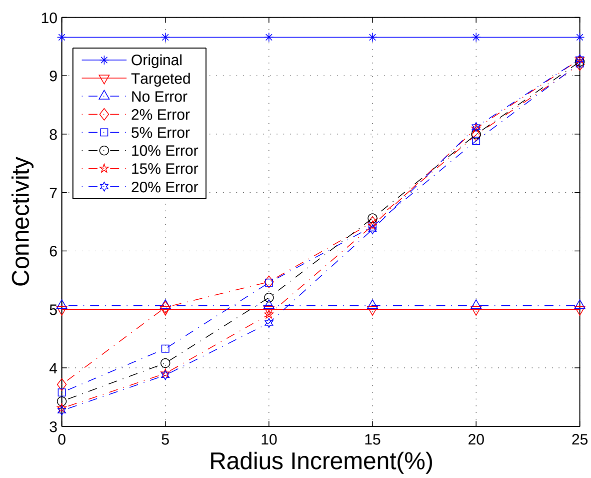
We considered fault-tolerant topology control algorithms in wireless ad hoc networks. We devised two algorithms: a centralized greedy algorithm, FGSS$_k$, and a localized algorithm, FLSS$_k$, and show that both



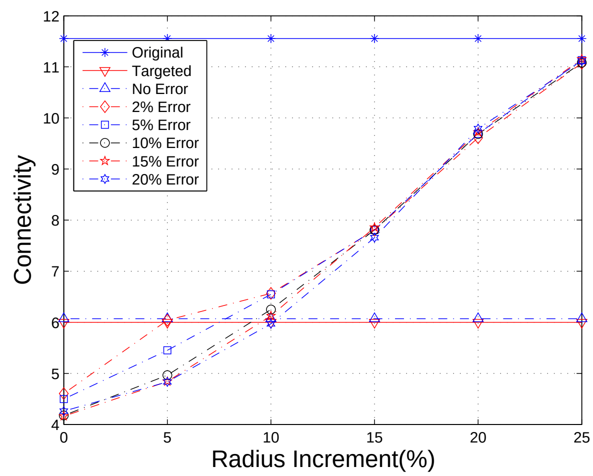
(a) $N = 30, K = 3$



(b) $N = 40, K = 4$



(c) $N = 50, K = 5$

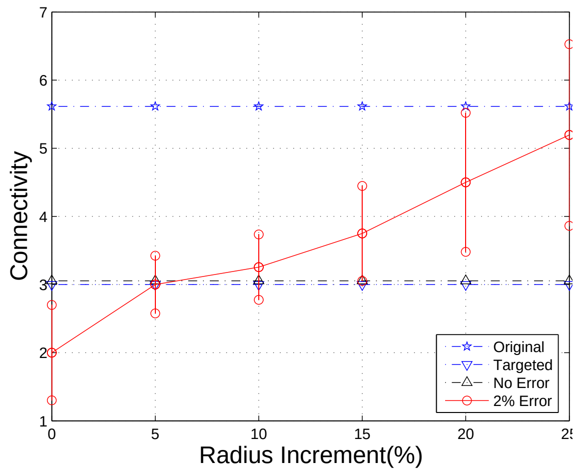


(d) $N = 60, K = 6$

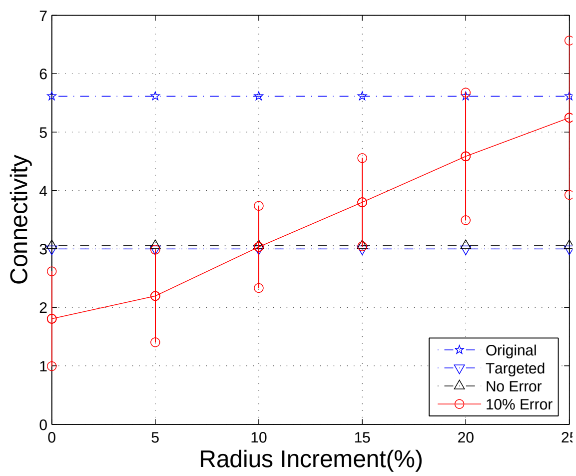
Figure 5.15: Robustness of FLSS$_k$ w.r.t. position estimation errors

algorithms preserve k-connectivity and are min-max optimal. We further examined several widely used assumptions in topology control, and discuss how to relax these assumptions. Simulation results indicated that FLSS$_k$ outperforms most distributed/localized fault-tolerance centric topology control algorithms with respect to out-degree, maximum radius, power consumption, energy efficiency, and network capacity.

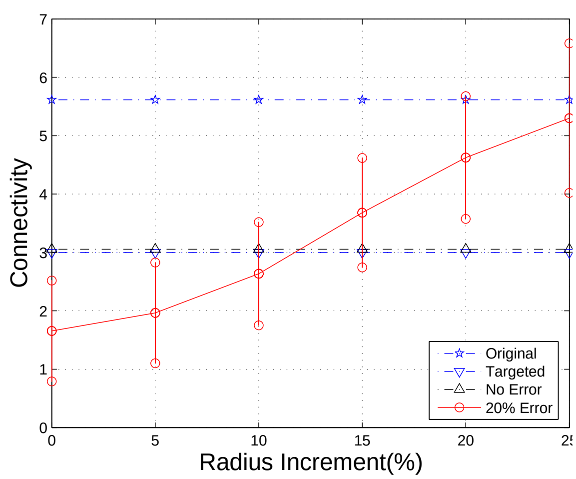
Although FLSS$_k$ outperforms other localized algorithms in random networks with respect to power consumption, we were unable to give any performance bound on power consumption as many centralized algorithms do [54]. In contrast, the distributed version of Hajiaghayi's algorithm [40] is shown to have a performance bound, but does not preserve k-connectivity. The dominating reason for the lack of a performance bound is that FLSS$_k$ is greedy and highly localized. Although it performs very well in most cases, we



(a) 2% error

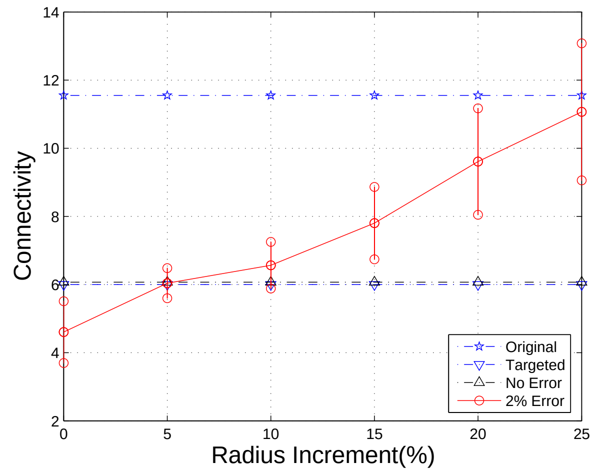


(b) 10% error

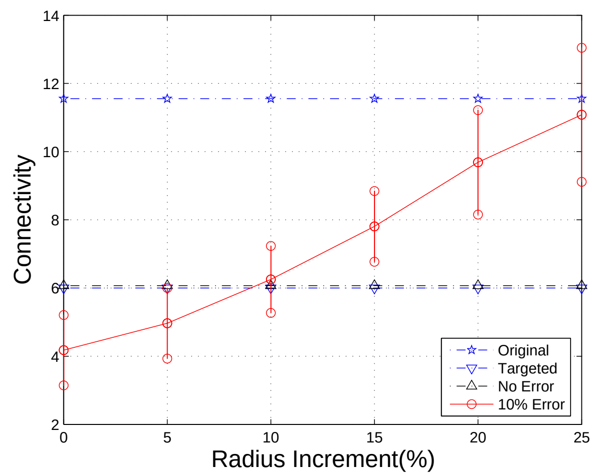


(c) 20% error

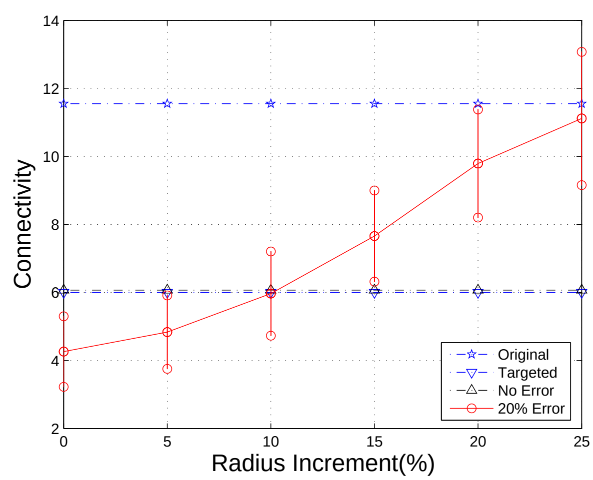
Figure 5.16: Average connectivity and standard deviation for $N = 30$



(a) 2% error



(b) 10% error



(c) 20% error

Figure 5.17: Average connectivity and standard deviation for $N = 60$

conjecture that the limited information of the visible neighborhood of a node is not sufficient to upper-bound the performance under some rare, extreme cases.

Chapter 6

Application of Topology Control on Power-Efficient Broadcast

Broadcast is a very important operation in wireless networks, as it provides an efficient way of communication that does not require global information and functions well in the case of changing network topologies. For example, many unicast routing protocols for wireless ad hoc networks [3, 50, 86, 89, 103] use broadcast in the stage of route discovery. Similarly, several information dissemination protocols in wireless sensor networks use some form of broadcast/multicast for solicitation and/or collection of sensor information [42, 47, 121]. Since wireless sensor networks mainly use broadcast for communication [2], how to deliver messages to all the wireless devices in a scalable and power-efficient manner has drawn more and more attention.

Although many broadcast/multicast algorithms have been proposed for wireless multi-hop networks [11, 34, 38, 49, 59, 82, 97, 117, 118], most of them are not power-aware. Design of power-efficient broadcast/multicast algorithms in wireless ad hoc networks is difficult due to the following reasons.

Firstly, wireless networks do not come with predefined "links". The level of transmission power determines the number of neighbors (and hence the scope of broadcast), which in turn determines the number of times a broadcast message has to be relayed to reach all nodes (and hence the delay).

Secondly, the trade-off between (i) using larger power and reaching more nodes in one hop and (ii) using smaller power and relaying broadcast messages through multiple intermediate nodes cannot be easily characterized. Even in the case that each node has unconstrained transmission power and can reach every other node in one hop (by using the maximal transmission power), it is proved in [1, 13, 76] that the minimum-energy broadcast tree problem is NP-complete. The problem is further complicated when the transmission power of each node is constrained, where heuristics devised under the unconstrained power assumption (e.g., BIP [115] and IMBM [60]) cannot be directly applied without modifications.

Thirdly, it is essential that the communication overhead in building a power-efficient broadcast tree be kept minimal, since the broadcast tree may have to be frequently rebuilt/adjusted as a result of topological changes. This is especially true for large-scale sensor networks where the topology is changing dynamically due to the changes of position, energy availability, environmental interference, and failures. Therefore, centralized algorithms requiring global information are not be practical.

Not until recently have research efforts been made to devise power-efficient multicast/broadcast algorithms for wireless networks. Several of these algorithms are centralized, and do not consider computational and message overheads incurred in collecting global information. Several of them also assume that the network topology does not change between two runs of information exchange. These assumptions may not hold in practice — the network topology may change from time to time, and the computation and energy overhead incurred in collecting necessary information is not be negligible, especially in the case of the existence of a large number of mobile devices.

We first investigate the design trade-off between (i) using higher transmission power to reach more nodes within one hop and (ii) using lower power and relaying broadcast messages through multiple hops. Our analysis shows that multi-hop broadcast is more power-efficient when $\alpha \geq 2.2$, where α is the path loss exponent in the radio propagation model $P(r, \alpha) = c_0 \cdot r^\alpha + c_1$, where r is the distance between the transmitter and the receiver. Therefore, each node should relay broadcast messages by using transmission power that is as small as possible, as long as it can ensure the delivery.

Based on the analysis, we then propose a power-efficient broadcast algorithm, *Broadcast over Local Spanning Subgraph* (BLSS). In BLSS, an underlying topology is first constructed by using FLSS [65] (Chapter 5), a localized topology control algorithm that can preserve k-connectivity of the network. The value of k can be varied depending on the requirement for fault tolerance. Then broadcast messages are simply relayed on the derived topology via constrained flooding. BLSS has several desirable features: (1) it does not rely on the assumption of unconstrained power or a specific power consumption model; (2) it requires only local (rather than global) information and adapts well to mobility; (3) it achieves 100 % broadcast coverage; and (4) it provides a reliable shared broadcast infrastructure.

The simulation results indicate that the performance of BLSS is comparable to that of a centralized algorithm, BIP [115]. If the communication overheads incurred in collecting all the necessary information are considered, BLSS outperforms BIP in large-scale wireless networks with modest mobility.

The rest of this chapter is organized as follows. We first summarize the existing work in Section 6.1. Then we study in Section 6.2 the trade-off between (i)using higher power to reach more nodes within one hop and (ii) using lower power and relaying broadcast messages through more hops. We propose BLSS in Section 6.3, and evaluate its performance in Section 6.4. Finally, we conclude this chapter in Section 6.5.

6.1 Related Work

Existing work can be categorized into the following six groups.

6.1.1 Power-aware Broadcast

Singh *et al.* [104] proposed five power-aware metrics to calculate broadcast routes, and showed via simulation that the use of power-aware metrics can reduce the energy consumption. However, as they simply used power-related metrics to construct broadcast/multicast trees, they did not consider how wireless devices can adjust their transmission power to further reduce power consumption and possibly increase the network capacity.

6.1.2 Flooding-based Schemes

Santiv*áñ*ez *et al.* [100] show that flooding is a good solution for the sake of scalability and simplicity. Several flooding techniques for wireless networks have been proposed [85, 92, 106], each with respect to certain optimization criteria. However, none of them take advantage of the feature that the transmission power of a node can be adjusted. Although our proposed approach is also based on constrained flooding, it further takes advantage of a node's ability to adjust its transmission power.

6.1.3 Optimization-based Schemes

The scheme proposed in [45] is built upon a search-based paradigm in which the minimum-cost broadcast/multicast tree is constructed by a search process. Two procedures are devised to check the viability of a solution in the search space. Preliminary experimental results show that this method renders better solutions than BIP, though at a higher computational cost.

Liang [76] showed that the minimum-energy broadcast tree problem is NP-complete, and proposed an approximate algorithm to provide a bounded performance guarantee for the problem in the general setting. Essentially the minimum-energy broadcast tree problem is reduced to an optimization problem on an auxiliary weighted graph, and the optimization problem is solved so as to give an approximate solution for the original problem. Another algorithm was also proposed, which yields better performance under a special case.

Das *et al.* [23] proposed an evolutionary approach using genetic algorithms. The same authors also presented in [24] three different integer programming models which can be used to find the solutions to the minimum-energy broadcast/multicast problem. The major drawback of optimization based schemes is, however, that they are centralized and require the availability of global topological information.

6.1.4 Centralized Heuristics

The *Broadcast Incremental Power* (BIP) algorithm [115] constructs energy-efficient, source-based broadcast/multicast trees in wireless ad hoc networks. It is assumed that each node may adjust its transmission power, and energy is consumed only during transmissions. To build a energy-efficient broadcast/multicast tree, the algorithm begins by determining the node that the source can reach with the smallest transmission power. At each step thereafter, one new node with the minimum additional cost is added to the tree. When the tree construction completes, an additional procedure called *sweep* is taken to eliminate unnecessary transmissions and further reduce the total energy consumption. BIP is probably the first algorithm to exploit the trade-off between reaching more nodes in a single hop by using larger power and relaying messages through multiple hops by using lower power. As analyzed in [112], BIP, like other greedy heuristics, may render "bad" results under certain scenarios.

The *Iterative Maximum-Branch Minimization* (IMBM) algorithm [60] was another effort to construct power-efficient broadcast trees. It starts with a basic broadcast tree in which the source directly transmits to all other nodes. Then it attempts to approximate the minimum-energy broadcast tree by iteratively replacing the maximum branch with less-power, more-hop alternatives.

Centralized heuristics have the following drawbacks. Firstly, they are usually devised under the assumption that the transmission power is unconstrained so that any two nodes can communicate with each other. This is not true in realistic wireless ad hoc networks where message relaying is usually needed. Secondly, they are for session-based broadcasting only, i.e., it is assumed that the network does not change during a session, and the computation and energy necessary in the stage of session establishment is negligible compared to that of the broadcasting. Thirdly, the communication overhead to collect and dispatch information is relatively high. The source node needs to know the position of (or the distance to) every other node, and each node needs to know its downstream, on-tree neighbors so as to correctly propagate broadcast messages. Fourthly, each source has to compute its own broadcast tree, which is sensitive to the power consumption model used.

6.1.5 Distributed Heuristics

Wieselthier *et al.* [116] proposed and evaluated two distributed versions of BIP: *Distributed-BIP-All* and *Distributed-BIP-Gateways*. In Distributed-BIP-All, each node calculates its local BIP tree and broadcasts the tree to all one-hop neighbors. Then each node forms the global tree by combining the local trees in certain temporal manner. The major drawback of Distributed-BIP-All is that many unnecessary transmissions are incurred when the global tree is constructed. Distributed-BIP-Gateways was proposed to overcome this disadvantage by selecting gateways to form local BIP trees that can reach all neighbors within two hops. After the global tree is constructed, the *sweep* operation (which is essentially a centralized procedure) can be applied. The performance of both distributed versions without the sweep operation is 30%-50% worse than that of BIP.

Cagali *et al.* [13] proposed the *Embedded Wireless Multicast Advantage* (EWMA) algorithm. EWMA uses the link-based minimum spanning tree (MST) as the initial solution and adjusts the transmission range for each node to reduce the total cost. A distributed version that involves construction of an MST in a decentralized manner was also proposed. Simulations show that both EWMA and its distributed version outperform BIP. The distributed version of EWMA, however, relies on global information that is multiple hops away to construct the MST, and may not perform well under frequent topological changes.

Ahluwalia *et al.* proposed a clustering-based algorithm that takes the following three steps [1]. A distributed clustering algorithm is first used to divide the entire network into clusters. Then a clustering sweep procedure is used to further reduce the transmission range. Finally a distributed MST construction algorithm is applied to join all the clusters together. Simulation results show its performance is 18% worse than that of BIP. This algorithm has the same drawback as EWMA, as it also relies on information that is multiple hops away to construct the MST.

All aforementioned approaches so far are subject to the power consumption model used, and the resulting broadcast tree might be totally different for slightly different models. This limits the applicability of those algorithms in realistic scenarios where the power consumption model may be time-varying. In addition, the broadcast is not fault-tolerant, since the broadcast structure is very close to a tree. If some nodes fail (as a result of power depletion, for example), there will be no guarantee that the message can be delivered to all other nodes in the network.

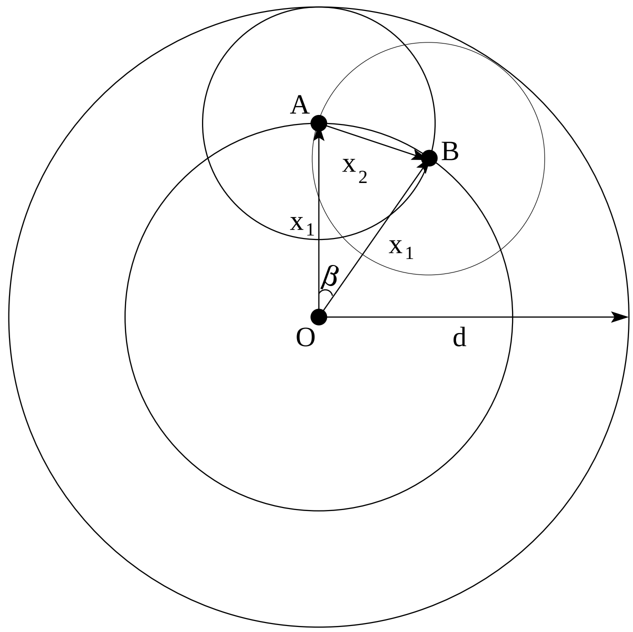
6.1.6 Localized Heuristics

A localized algorithm depends only on local information and is thus more robust to topological changes. Our work falls in this category. Cartigny *et al.* [16] proposed a localized algorithm, RBOP, which is built upon RNG and comes closest to our work. In RBOP, the broadcast is initiated at the source and propagated, following the rules of neighbor elimination, on the topology represented by RNG. Simulation results show that the performance degradation could be as high as 100% as compared to BIP.

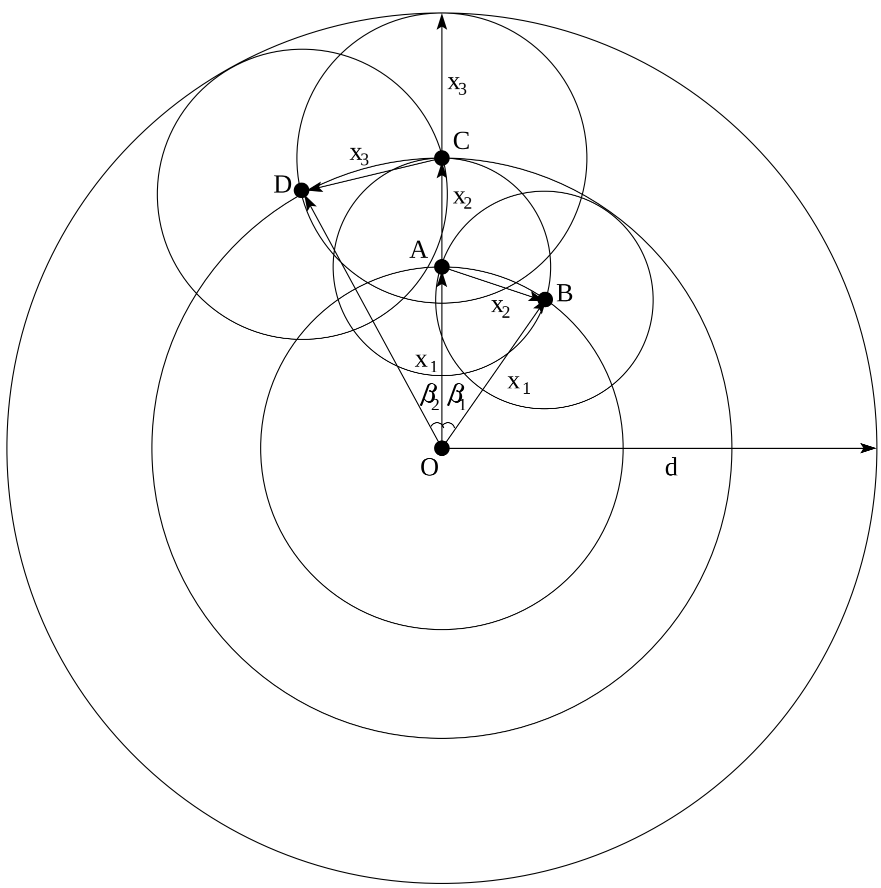
6.2 Relay or Not Relay

One of the fundamental issues for devising power-efficient broadcast algorithms in wireless ad hoc networks is to determine whether or not it is advantageous to use smaller power and relay messages through several hops. In this section, we investigate this problem under a model that can capture all the essential features of power consumption, message relay, and coverage. Based on the findings, we will then devise a localized broadcast algorithm.

Specifically, we consider the model where all nodes in the network reside inside a disk D of radius d, and focus on the power consumption incurred in the broadcast from the node residing at the center of the disk. We assume that (**A1**) the density of nodes is large enough so that a relay node can be found inside the disk whenever necessary; (**A2**) the area covered by a transmission is the disk centered at the transmitter; (**A3**) the power consumption model is $P(r, \alpha) = c_0 \cdot r^{\alpha} + c_1$, where r is the distance between the transmitter and the receiver, and $2 \leq \alpha \leq 4$ (here let $c_0 = 1, c_1 = 0$ for simplicity); and (**A4**) there is no constraint on the transmission power, i.e., a node can reach any other node in the network. Note that in a large-scale network of high node density, (**A1**) approximately holds. A broadcast is complete when the union of the



(a) Two-hop broadcast: $x_1 + x_2 = d, 0 \leq x_2 \leq x_1 \leq d$



(b) Three-hop broadcast: $x_1 + x_2 + x_3 = d, 0 \leq x_3 \leq x_2 \leq x_1 \leq d$

Figure 6.1: Two-hop broadcast and three-hop broadcast

disks centered at all transmitters can approximately cover the disk D. With the above model, we would like to determine whether one-hop broadcast or multi-hop broadcast is more power-efficient.

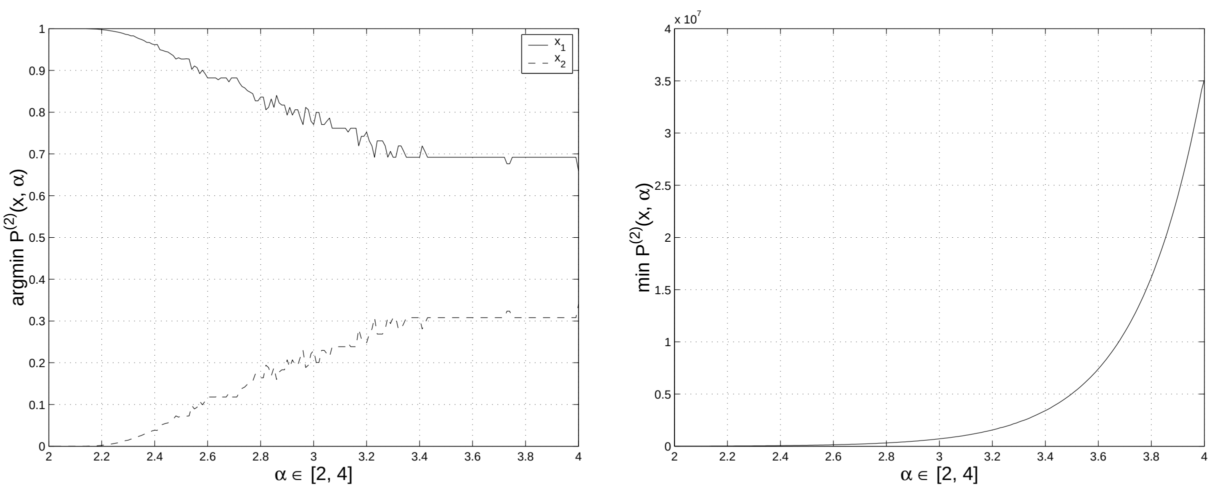
One-hop versus two-hop As shown in Figure 6.1(a), in the case of two-hop broadcast, node O first sends the message to all nodes within distance x_1, then nodes at the distance x_1 from node O are further selected to relay the message to all the nodes within distance x_2. x_1 and x_2 satisfy:

$$x_1 + x_2 = d, \quad 0 \le x_2 \le x_1 \le d, \tag{6.1}$$

$$\beta = \arccos \frac{2x_1^2 - x_2^2}{2x_1^2}, \tag{6.2}$$

and the total power, $P^{(2)}(\vec{x}, \alpha)$, consumed under two-hop broadcast is

$$P^{(2)}(\vec{x}, \alpha) = x_1^\alpha + \left\lceil \frac{2\pi}{\beta} \right\rceil x_2^\alpha. \tag{6.3}$$



(a) Optimal transmission ranges $\arg\min_{\vec{x}} P^{(2)}(\vec{x}, \alpha)$ (in terms of the ratio to d) as a function of α

(b) Minimum total power consumption $\min P^{(2)}(\vec{x}, \alpha)$ as a function of α

Figure 6.2: The optima in broadcast of up to 2 hops

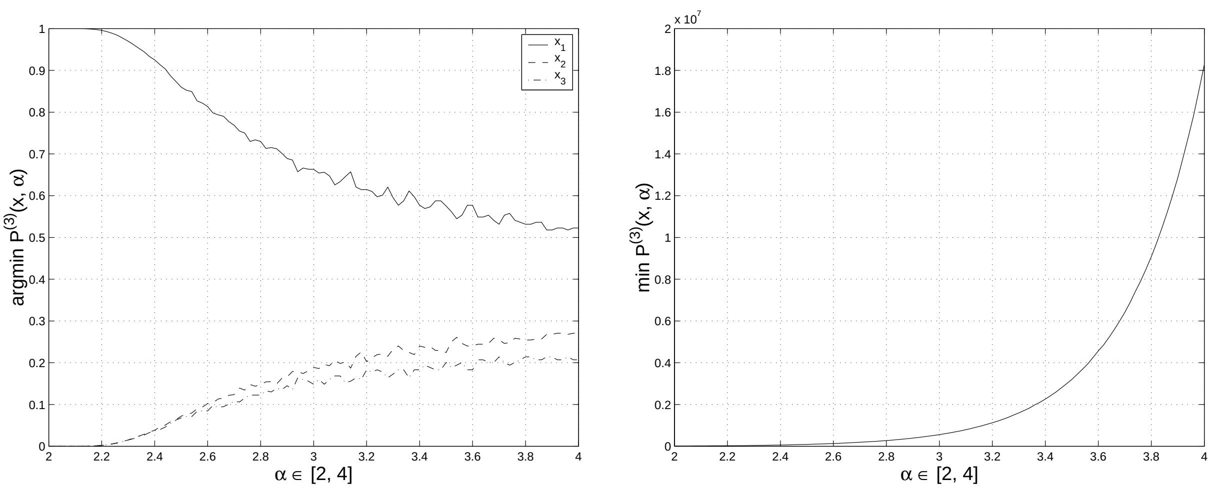
We obtain the minimum value of $P^{(2)}(\vec{x}, \alpha)$ numerically for $d = 100$. As shown in Figure 6.2, it is more desirable to use one-hop broadcast if $2 \le \alpha < 2.2$ (in which case $x_1 = d$ and $x_2 = 0$), and two-hop broadcast if $\alpha \ge 2.2$ (the optimal values of $\vec{x}$ are also given in Figure 6.2). Note that the total power consumed for small values of α is not zero, but is rather small as compared with that for large values of α.

One-hop versus multi-hop broadcast The above scenario can be straightforwardly extended to broadcasts of up to 3 hops. As shown in Figure 6.1(b), x_1, x_2, x_3 satisfy

$$\begin{aligned} x_1 + x_2 + x_3 &= d, \;\; 0 \le x_3 \le x_2 \le x_1 \le d, && (6.4) \\ \beta_1 &= \arccos \frac{2x_1^2 - x_2^2}{2x_1^2}, && (6.5) \\ \beta_2 &= \arccos \frac{2(x_1+x_2)^2 - x_3^2}{2(x_1+x_2)^2}, && (6.6) \end{aligned}$$

and power consumption function, $P^{(3)}(\vec{x}, \alpha)$, under three-hop broadcast is

$$P^{(3)}(\vec{x}, \alpha) = x_1^\alpha + \left\lceil \frac{2\pi}{\beta_1} \right\rceil x_2^\alpha + \left\lceil \frac{2\pi}{\beta_2} \right\rceil x_3^\alpha. \qquad (6.7)$$



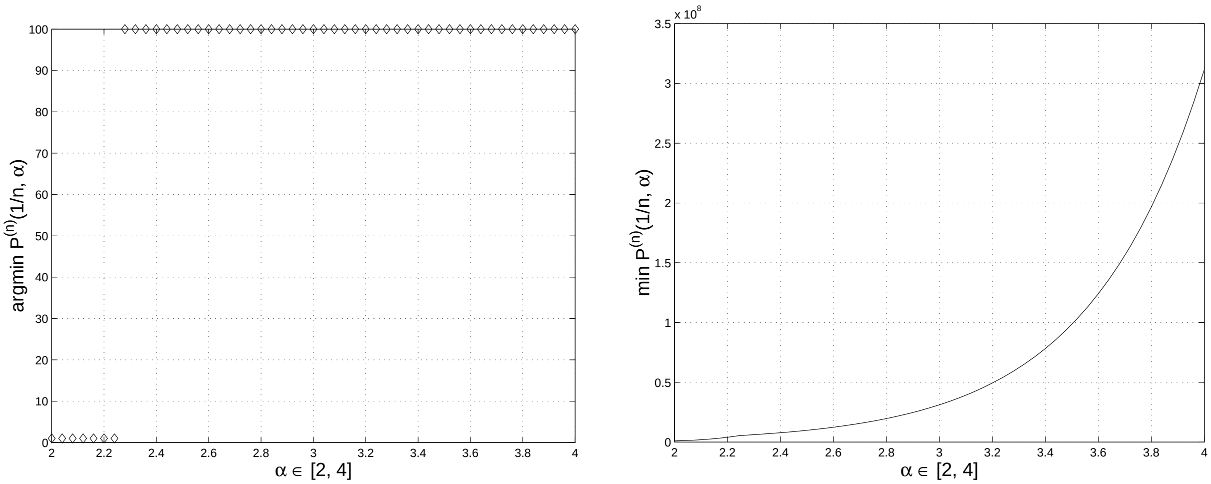
(a) Optimal transmission ranges $\arg\min_{\vec{x}} P^{(3)}(\vec{x}, \alpha)$ (in terms of the ratio to d) as a function of α

(b) Minimum total power consumption $\min P^{(3)}(\vec{x}, \alpha)$ as a function of α

Figure 6.3: The optima in broadcast of up to 3 hops

Figure 6.3 gives the optimal values of $\vec{x}$ and the minimum total power consumed in the case of broadcast of up to 3 hops (for $d = 100$). One-hop broadcast is still the most power-efficient when $\alpha < 2.2$, while multi-hop broadcast with optimal values of x outperforms when $\alpha \ge 2.2$.

Note that in the above formulation (Equation 6.4 - Equation 6.6), one-hop broadcast is simply a special case of the multi-hop broadcast where $x_1 = 1$ and $x_i = 0$ for $i \ge 2$. As we obtain similar results for broadcast that allows up to 16 hops, we conjecture that one-hop broadcast (multi-hop broadcast) is optimal when $\alpha < 2.2$ ($\alpha \ge 2.2$).



(a) Optimal value of n ($\arg\min_n P^{(n)}(\frac{1}{n}, \alpha)$) as a function of α

(b) Minimum total power consumption $\min P^{(n)}(\frac{1}{n}, \alpha)$ as a function of α

Figure 6.4: The optima in n-hop broadcast ($1 \leq n \leq 100$)

An alternative formulation of the problem is to assume an n-hop broadcast with every hop using the same power level. The results for the case of $n \leq 100$ is shown in Figure 6.4. Again we reach a similar conclusion that for $2 \leq \alpha < 2.2$ ($\alpha \geq 2.2$), one-hop broadcast (100-hop broadcast) is more power-efficient. Note that the optimal value of n is either 1 or 100, which suggests that it is more efficient to either use the largest power (when $\alpha < 2.2$) or the smallest power (when $\alpha \geq 2.2$).

So far we have used the power consumption model of the form $P(r, \alpha) = c_0 \cdot r^\alpha + c_1$, where $c_0 = 1, c_1 = 0$. For the case of $c_1 > 0$, we observe similar results, except that the threshold may be a little bit higher. Note, however, that the threshold difference in the two models is very small since c_1 remains a fixed cost that is relatively small compared with $c_0 \cdot r^\alpha$, which grows exponentially with α.

6.3 BLSS Algorithm

As analyzed in the first formulation in Section 6.2, multi-hop broadcast is optimal for $\alpha \geq 2.2$ in the case of unconstrained transmission power. The second formulation also suggests that if $\alpha \geq 2.2$, it is more efficient for each node to use smaller power. Given the fact that α is usually larger than 2 in reality [95], we propose a multi-hop broadcast algorithm based on constrained flooding.

BLSS (Broadcast over Local Spanning Subgraph) is a constrained flooding algorithm applied to the network topology generated by FLSS, with one refinement that a node will not relay the message if it has

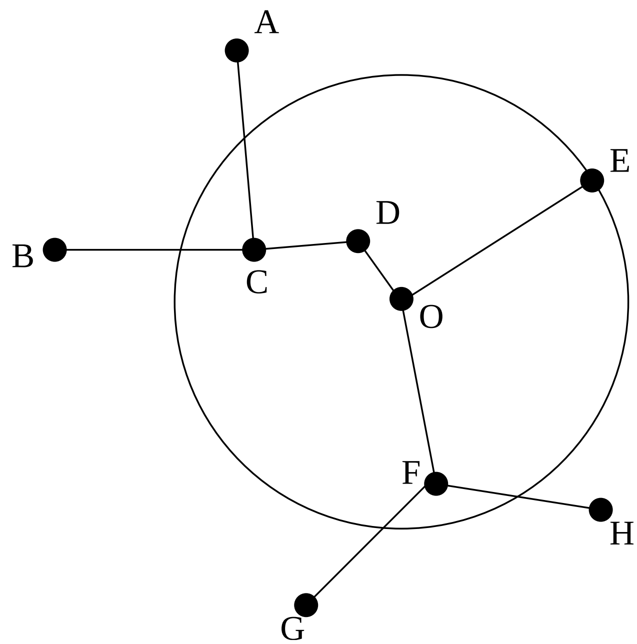


Figure 6.5: An example of BLSS

received a broadcast message from all its neighbors (in the FLSS-induced topology) or has known that all its neighbors have received the message by overhearing. This reduces the extent to which messages are flooded. For example, in Figure 6.5, node O is the source node with neighbors D, E, and F. Node D will not relay broadcast messages from node O since all its neighbors (nodes C and O) have received the message. The reason for flooding on the FLSS-induced network topology is that, according to our analysis in Section 6.2, it is more power-efficient for each relay node to use smaller transmission power. As we have proved that FLSS is min-max optimal, the largest radius any node has in the FLSS-induced topology (or equivalently the largest transmission power a node will use) is minimized.

Here we only applied a simple optimization rule to suppress the number of relay nodes. There are several methods available to suppress the number of relay nodes in the context of mitigating the broadcast storm problem [82, 117] (note, however, that most of them do not consider power control). These techniques can be straightforwardly applied, but will only lead to marginal improvement, as the minimally connected topology generated by FLSS has already significantly reduced unnecessary relays.

BLSS has several desirable features. Firstly, by virtue of the fact that the correctness of FLSS does not rely on a specific power consumption model, BLSS is independent of the underlying power consumption model. Secondly, since BLSS requires only local information, the power consumed due to exchange of control information is not as significant as in algorithms that rely on global topological information. Consequently, BLSS is scalable and adapts well to topology changes (due to node failure or mobility). Thirdly, since FLSS preserves k-vertex-connectivity, BLSS is $k-1$ fault-tolerant, i.e., the failure of at most $k-1$ nodes will not disconnect the network.

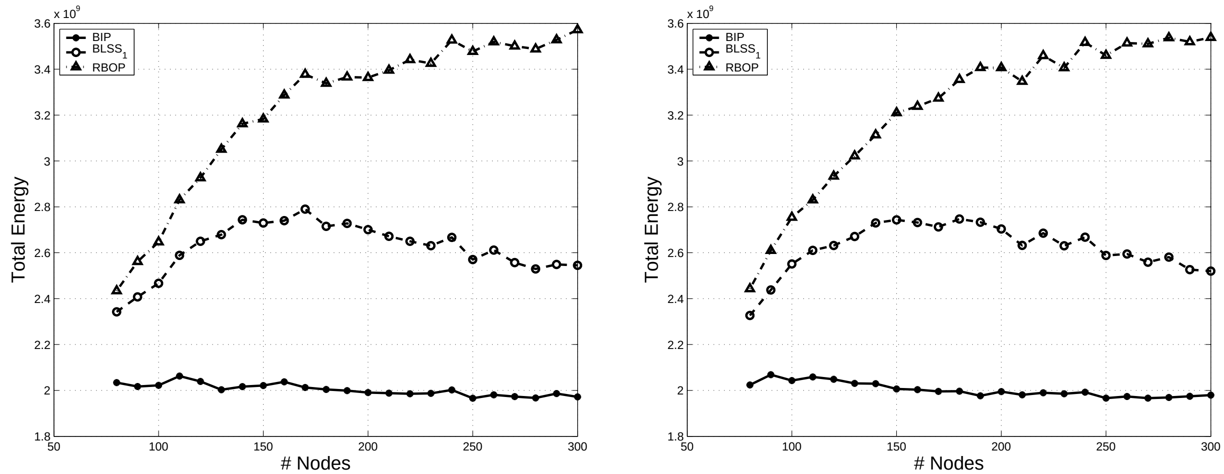
6.4 Performance Evaluation

In this section, we evaluate the performance of BLSS. Let BLSS_k denote the algorithm that is $k-1$ fault-tolerant. We first compare BLSS with two representative power-efficient broadcast algorithms: BIP and RBOB. Since BIP and RBOB do not provide fault tolerance, we only compare them against BLSS_1. As mentioned in Section 6.1, BIP is perhaps the first (centralized) broadcast algorithm that addresses the design trade-off of relaying versus not relaying, and has been often used as a baseline algorithm for comparison. RBOP falls in the category of localized heuristics (Section 6.1) and comes closest to our work.

The reasons for not to compare BLSS against other distributed heuristics such as *Distributed-BIP-All*, *Distributed-BIP-Gateways*, EWMA, and clustering-based algorithm are two-fold: (1) they require an accurate power consumption model to construct the broadcast tree; and (2) they rely on information that is multiple hops away to construct the broadcast tree, and the overhead incurred in collecting the information often outweighs the benefit.

Two radio propagation models, *Free Space Model* and *Two-Ray Ground Model*, are used in the simulation. In the first set of simulations, n nodes are uniformly distributed inside a disk, where n varies from 80 to 300. The radius of the disk is $1000m$ under the Free Space Model and $400m$ under the Two-Ray Ground Model. The transmission range is $260m$ under the Free Space Model and $100m$ under the Two-Ray Ground Model. One node is randomly selected to be the source of broadcasting. In the second set of simulations, nodes are distributed in a clustered fashion. 5 sub-disks of radius $100m$ are first randomly distributed inside the disk; then n nodes are uniformly distributed inside each sub-disk, where n varies from 15 to 60. Each data point is the average results of 50 simulation runs.

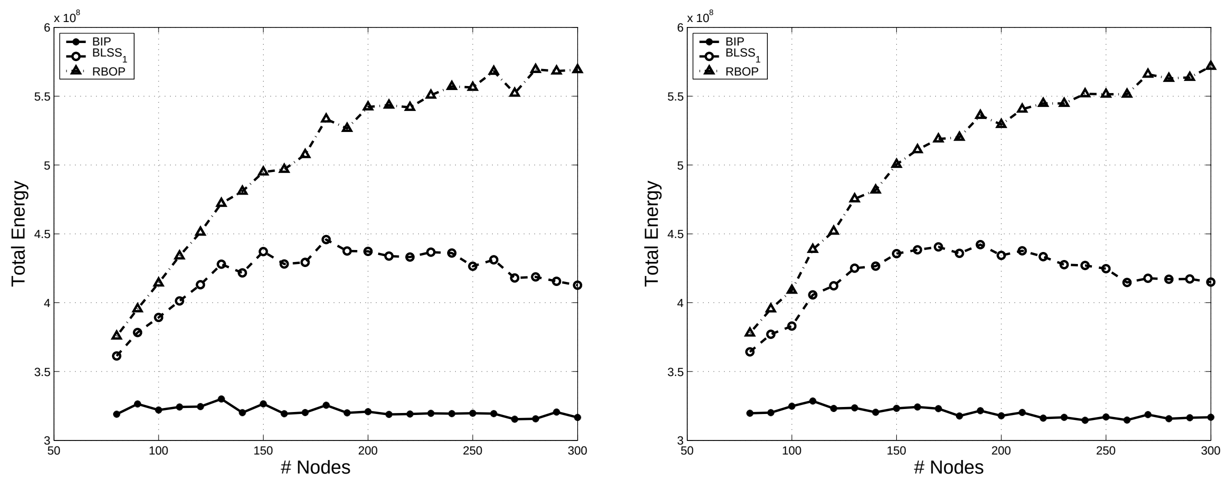
Figure 6.6 and Figure 6.7 show the total power consumed by BIP, RBOP, and BLSS under different propagation models, for uniform node distribution. And Figure 6.8 and Figure 6.9 show the total power consumed by BIP, RBOP, and BLSS under different propagation models, for clustered node distribution. BIP outperforms localized algorithms, since it can better optimize its performance by utilizing the global



(a) Free Space Model, no fixed cost ($c_1 = 0$)

(b) Free Space Model, fixed cost $c_1 = 1000$

Figure 6.6: Performance comparison under uniform distribution and Free Space Model



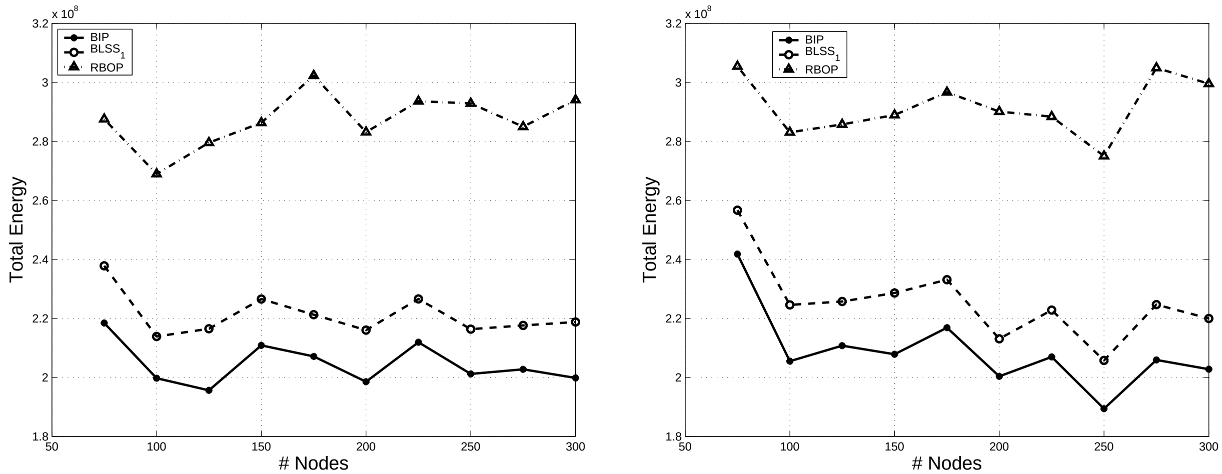
(a) Two-Ray Ground Model, no fixed cost ($c_1 = 0$)

(b) Two-Ray Ground Model, fixed cost $c_1 = 1000$

Figure 6.7: Performance comparison under uniform distribution and Two-Ray Ground Model

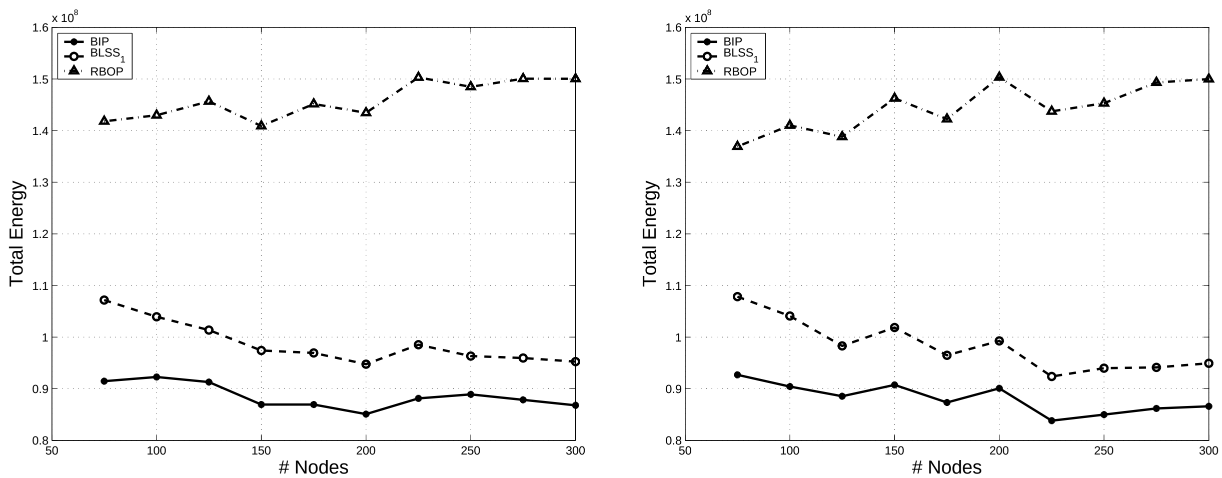
information. The performance of BLSS is much better than that of RBOP, and is comparable to that of BIP in the case of clustered node distribution. BLSS outperforms RBOP by virtue of the min-max optimality of the topology induced by FLSS. In the case of uniform node distribution, BLSS is 13%-39% worse than BIP, while RBOP is 18%-81% worse. In the case of clustered distribution, BLSS is 6%-17% worse than BIP, while RBOP is 26%-73% worse.

Figure 6.10 compares the average physical out-degree and average radius of FLSS (the underlying topology of BLSS) and RNG (the underlying topology of RBOP). The maximal transmission range is $260m$. It is clear that nodes in FLSS_1 generally have lower out-degrees and shorter radii. Moreover, the average



(a) Free Space Model, no fixed cost ($c_1 = 0$) (b) Free Space Model, fixed cost $c_1 = 1000$

Figure 6.8: Performance comparison under clustered distribution and Free Space Model

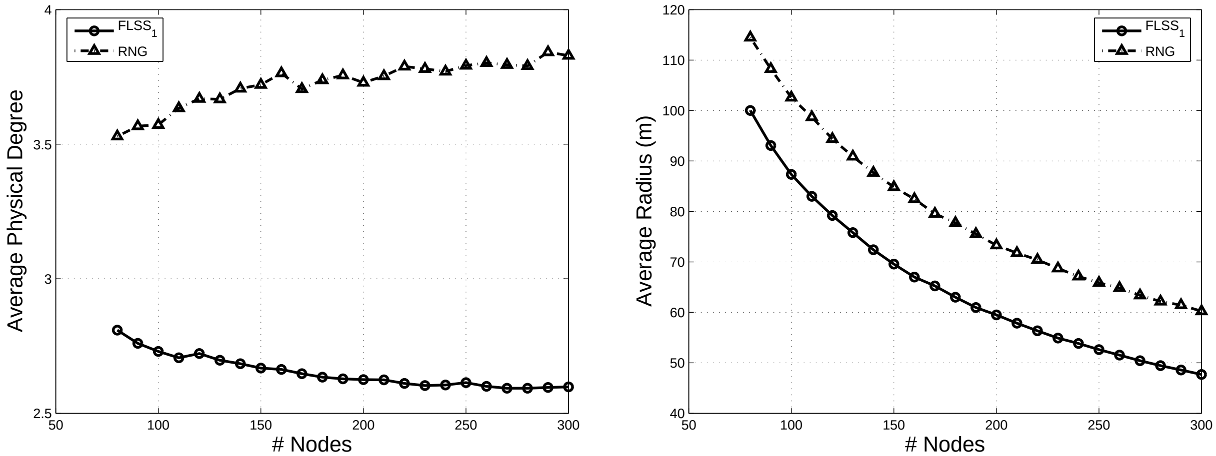


(a) Two-Ray Ground Model, no fixed cost ($c_1 = 0$) (b) Two-Ray Ground Model, fixed cost $c_1 = 1000$

Figure 6.9: Performance comparison under clustered distribution and Two-Ray Ground Model

degree of FLSS$_1$ is is relatively stable with regard to the network size, which ensures that BLSS is scalable.

Note that in the above study, we ignore the power consumed to collect global information in BIP. Now we look into this issue more carefully. Let P_{info}^{bip} and P_{info}^{blss} denote the power consumed in the stage of information collection required by BIP and BLSS, respectively, and let P_{trans}^{bip} and P_{trans}^{blss} denote the total transmission power incurred in a broadcast session under BIP and BLSS, respectively. Under the assumption that m broadcasts take place between two runs of information exchange, the total power consumed between



(a) Average physical out-degree

(b) Average radius

Figure 6.10: Comparison of $FLSS_1$ and RNG

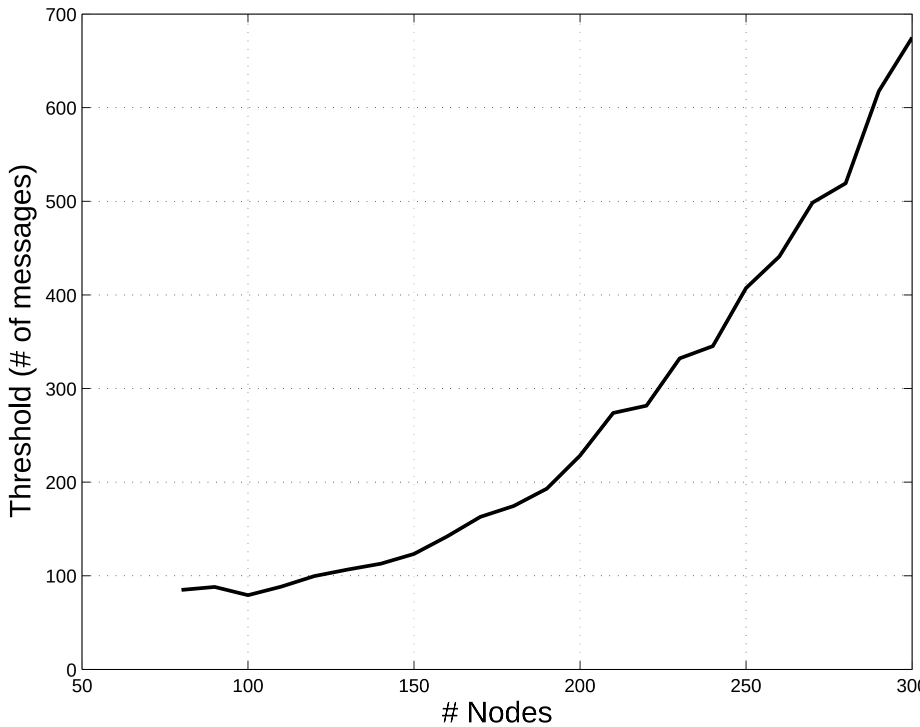
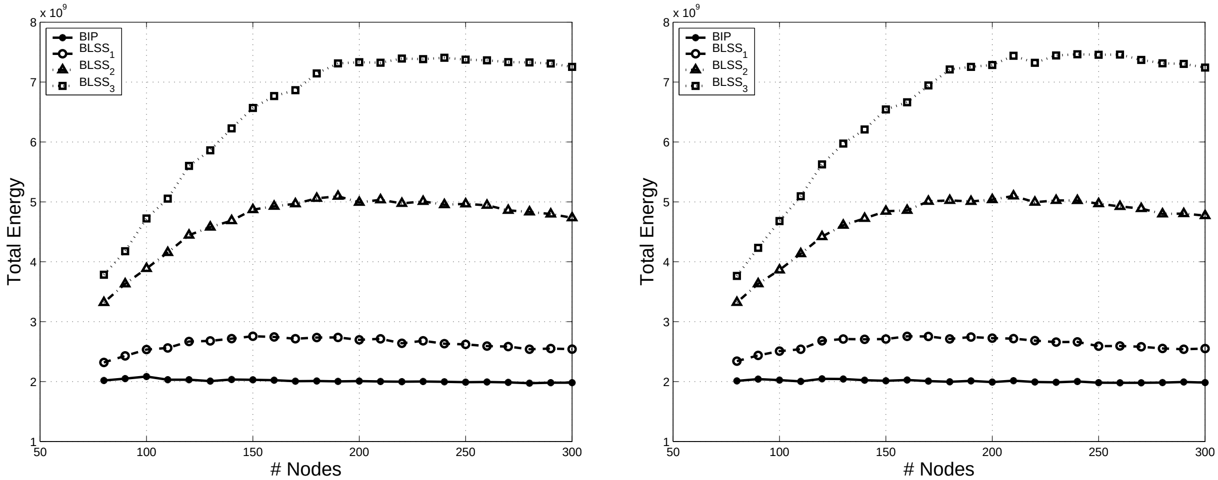


Figure 6.11: Number of broadcast packets needed for BIP to outperform BLSS

two information exchanges is

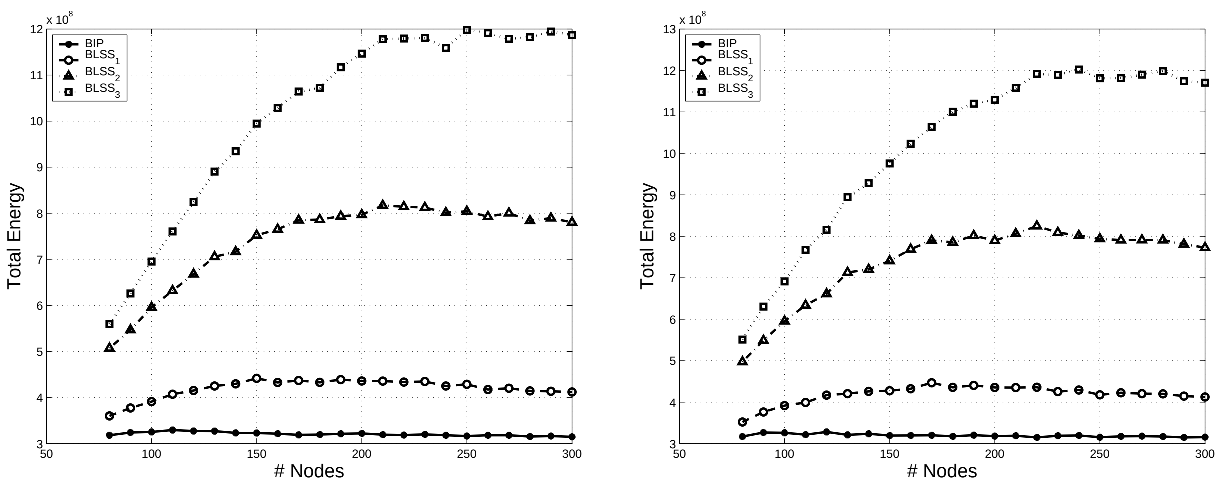
$$P_{total}^{bip} = P_{info}^{bip} + m \cdot P_{trans}^{bip}, \tag{6.8}$$

$$P_{total}^{blss} = P_{info}^{blss} + m \cdot P_{trans}^{blss}. \tag{6.9}$$



(a) Free Space Model, no fixed cost ($c_1 = 0$) (b) Free Space Model, fixed cost $c_1 = 1000$

Figure 6.12: Comparison of BIP and BLSS$_k$ (k=1,2,3) under Free Space Model



(a) Two-Ray Ground Model, no fixed cost ($c_1 = 0$) (b) Two-Ray Ground Model, fixed cost $c_1 = 1000$

Figure 6.13: Comparison of BIP and BLSS$_k$ (k=1,2,3) under Two-Ray Ground Model

Although $P^{bip}_{trans} < P^{blss}_{trans}$ (Figure 6.6–Figure 6.9), P^{bip}_{total} may still be larger than P^{blss}_{total} if the number of nodes, n, in the network is large, since $P^{bip}_{info} = O(n^2)$ and $P^{blss}_{info} = O(n)$. Only when m is large enough can BIP outperforms BLSS. To better investigate along this direction, we perform a simple analysis on m, the number of broadcasts required between two runs of information exchanges, for BIP to outperform BLSS in a network of size n.

Suppose UDP packets are used in broadcasts in an IEEE 802.11-operated network. The size of an control packet (for information exchange) is $40+18+6+30+4 = 98$ bytes (the first 40 bytes accounts for the UDP header and the IP header), and that of a broadcast packet is $40 + 18 + 6 + 2342 + 4 = 2410$ bytes. Under the assumption that power consumed in transmission is proportional to the amount of data transmitted, and let P_{unit} denote the power consumed for one unit of transmission, we have

$$P^{bip}_{info} = \frac{98}{2410} n^2 P_{unit}, \tag{6.10}$$

$$P^{blss}_{info} = \frac{98}{2410} n P_{unit}. \tag{6.11}$$

Only when m exceeds a certain threshold will the inequality $P^{bip}_{total} \leq P^{blss}_{total}$ hold. Figure 6.11 shows the threshold versus the network size n, under the Free Space model with a fixed cost c=1000. In a network of 300 nodes, it takes more than 600 broadcast packets between two information exchanges for BIP to outperform BLSS. In the case of high mobility, the network topology may have changed (thus invalidating the broadcast tree) before that many broadcasts have taken place.

In the last set of simulations, we investigate the trade-off between the degree of fault tolerance and power-efficiency. In particular, we compare the total power consumed by BLSS_1, BLSS_2, and BLSS_3 under the uniform node distribution using different propagation models. As shown in Figure 6.12 and Figure 6.13, a higher degree of fault tolerance requires higher power consumption.

6.5 Conclusions

In this chapter, we presented BLSS, a scalable, fault-tolerant algorithm for power-efficient broadcast for wireless ad hoc networks. Succinctly, an underlying topology is first derived by using FLSS, a localized fault-tolerant topology control algorithm (see Chapter 5). Then broadcast messages are simply relayed on the derived topology in a constrained flooding fashion. BLSS possesses several desirable features: (1) it does not rely on the assumption of unconstrained power or a specific power consumption model; (2) it requires only local (rather than global) information and adapts well to mobility; (3) it ensures 100 % broadcast coverage as long as the data transport is reliable; and (4) it provides a reliable shared broadcast infrastructure. Simulation results show that the performance of BLSS is comparable to that of BIP, a representative centralized algorithm, and is superior to that of RBOP, a representative localized algorithm.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

Topology control has crucial impact on the system performance of wireless ad hoc networks with regard to energy efficiency, network capacity, and end-to-end delay. In this thesis, we have designed, analyzed and proved many desirable properties of several localized topology control algorithms.

Specifically, for homogeneous wireless ad hoc networks where devices have uniform transmission ranges, we have proposed LMST, a fully localized topology control algorithm that is simple and efficient. We have also proposed two localized algorithm, DRNG and DLSS, for heterogeneous wireless ad hoc networks with nonuniform transmission ranges. This is one of the first efforts to address the connectivity and bi-directionality issues in heterogeneous wireless networks. To further improve the reliability of the network topology, we have proposed a localized algorithm, FLSS, that preserves k-vertex connectivity of the network and is min-max optimal among all the strictly localized algorithms. Finally, we devised, on top of FLSS, a reliable, power-efficient broadcast algorithm ,BLSS, for wireless networks. Simulation results have shown that our proposed algorithms are simple and scalable, and outperform existing algorithms in terms of energy efficiency, network capacity, and end-to-end delay.

7.2 Future Work

We have identified several research avenues for future work:

Dynamic Topology Control w.r.t. Network Traffic Existing topology control algorithms are mostly static, i.e., the transmission power of each node is determined only by geometric information such as positions of nodes. It will be more realistic to also take into account the network traffic dynamics and adjust the topology accordingly. For example, nodes with high volume of traffic should be assigned as low power as possible so as to save energy. On the other hand, nodes with no traffic do not need to be connected. The major challenge is how to collect traffic distribution information (which changes dynamically) and figure it into topology control decisions.

Incorporating Physical Layer Characteristics The UDG model [19] is widely used to characterize the transmissions in a wireless ad hoc network. Although the UDG model is good for deriving strong theoretical results, it does not capture the probabilistic apsect of wireless communication. The Quasi UDG [58], which only contains edges shorter than d ($0 \leq d \leq 1$), is one step toward a more realistic characterization of wireless communication. Still, we need a probabilistic model that can account for the fact that the transmission range of a node may not be the same in all directions and may be a time-varying function as a result of physical layer characteristics such as shadowing, multi-path fading and directional antennas.

Effect of MAC-layer Interference on Network Topology Most topology control algorithms assume a perfect MAC protocol so that contention can be ignored. It would be interesting to study (1) how the MAC-layer behaviors will affect network connectivity; and (2) how to use the topology information to reduce MAC-layer contention/collision.

Capacity of Wireless Networks with Topology Control It is generally conceived that topology control algorithms can increase network capacity, which has been corroborated by several simulation studies. However, a theoretical analysis that quantifies the order of magnitude of improvement has been lacking. An analysis that gives the per-session throughput with topology control exercised complete the entire spectrum of capacity analysis (Section 2.1.2).

Cross-Layer Design for Topology Control Previous research work indicates that topology control actually spans from the physical layer, the MAC layer, to the network layer. The tasks remain to understand the interaction between topology control and these layers, and to integrate topology control into the existing protocol stack. Both the architecture and the implementation methodology should be carefully studied for eventual deployment of topology control.

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